Introduction

The country's economical development and urbanization process lead to the growth of large cities and to concentration of most population in them. As the result there are great traffic problems in the largest cities caused by:

- rapid growth of the cars number and other vehicles in the cities;
- cities expansion plans realization, which leads to residential areas remoteness from the work places and city centers;
- insufficient and imperfect traffic system and land transport facilities;
- increase in passenger traffic flow due to the city's high pace of development and its population growth (Bannister & Raymond, 1984).

The construction of underground transport structures such as subway is an effective solution in terms of traffic system problems. Public transport system made up
from the joint usage of subways and land transport facilities is considered to be the most effective way of ensuring regular passengers transportation with a population over 1 million inhabitants (Clark, 1972; Edward & Gracie, 1981; Hurst, 1963; Lohwaters, 1990; Megaw & Bartlett, 1983).

Subways are complex and responsible engineering constructions, the engineering, construction and maintenance of which requires the solution of interconnected scientific and engineering tasks with the help of well-coordinated work of various professionals including the geodesists (Morozov, 1979; Ganshin, Storozhenko & Budenkov, 1991; Ustavich, Shulsky & Vinokurova, 2003). Geodesic support of the construction process is one of the most important and responsible parts of construction and installation process when building large underground structures (Chin et al., 2012; Ustavich, 2005; Iredale, 2010; Xu, 2010).

Geodesic support of subway construction in great cities has a set of peculiar features. The combination of geodetic control network with high standards is one of important feature/factor. Subway lines are developing through the time and it creates additional requirements to the stability of geodetic control network points, which imposes additional difficulties in a modern city life (Fedoseyev, 2010). Another considerable feature is that the geodetic control network are created and developed on the land surface, while the main building is conducted under the ground at a considerable depth. It is needed to consider the peculiarities of geodetic works while using the most effective modern measuring instruments, for example satellite, especially when processing the satellite measurements results which accompany the construction of the subway (Gairabekov & Kravchuk, 2010; Klyushyn & Dinh, 2013, Klyushyn & Mihelev, 2010).

Materials and Methods

The modern satellite measuring instruments provide a RMS error of the coordinates deviation in 5-10 mm at the distances usually found at the construction of the subway (Gairabekov & Kravchuk, 2010; Gairabekov, Naumenko & Yakovlev, 2009). However, this accuracy refers to the coordinates deviation in the cartesian reference system PZ-90 (PZ-90) when a satellite navigation system GLONASS is used, or in WGS-84 if the NAVSTARGPS system is used.

The peculiarity of using satellite measuring results in geodesy lies in the fact that it uses unique coordinate systems unlike the mathematics, physics or any other science connected with the linear quantity reference system. As a rule, in geodesy there is no collaborative coordinate system which was implemented in satellite navigation systems. There is a two-dimensional coordinate system in geodesy used for determining the planned position of the object (the latitude B and the longitude L concerning the reference ellipsoid), or also it could be its analogy in the Gauss projection coordinates, which is a curvilinear coordinate system, where the meridian arc length lies on the axis of abscissas and the arc from the central meridian to the defined point lies on the axis of ordinates. The third coordinate — the normal height — has almost no connection with the coordinate origin (with the geocenter), and is measured from its surface — the quasigeoid. It is historically stated due to the human practical activities to consider on real, but not flat surface of Earth that point lower to which the water flows (Zelensky & Dorofeeva, 1973; Runov, 1973; Fedoseyev, 1977). Consequently, the normal height is more of a physical quantity, than of the geometrical, and is inextricably connected with the Earth’s gravity. Such a unique composite system of coordinates designed by great mathematicians and geodesists provides an effective solution of the vast majority of geodesic tasks. However, in engineering geodesy it is
important to consider carefully the specifics of used coordinate systems when performing highly-precise works.

The widely used term "the coordinates of point" fully refers to the system of ellipsoidal coordinates (the latitude, the longitude, and the altitude), since all the points of normal are of equal importance of latitude and longitude to the ellipsoid. As for the Gauss projection coordinates the term "the coordinates of point" is weak, since they refer to the coordinates of point projection on the reference surface (point A, Fig. 1).

**Figure 1.** The character of the Gauss projection coordinates and the position of the point projection on the ellipsoid surface

Such feature of this coordinate system allows the quantity $S$, which is close to the geodesic line length on the ellipsoid surface, 

$$S = \sqrt{\Delta x^2 + \Delta y^2},$$  

(1)

to be calculated using the computation formula for the distance between the points, faithful for flat figures, only if the points are removed from the central meridian not more than for 20-30 km (until the deformations achieve considerable quantities).

As usual, real buildings are built at different geodesic heights concerning the reference ellipsoid. This leads to the fact that the horizontal distance on the surface of the construction site, calculated on the basis of the measurements, could not coincide with the distance computational results according to the formula (1) on a significant amount

$$\Delta S = \frac{HS}{R}$$  

(2)

where $H$ — the geodesic height of the object; $S$ — the horizontal distance; $R$ — Earth’s radius.

When analyzing the results of geodetic measurements in Moscow one should be especially responsible and careful, since the F.W. Bessel (1844) ellipsoid is used as a reference ellipsoid instead of more precise F.N. Krasovsky (1928) ellipsoid (Table 1).

**Table 1. Ellipsoid’s parameters**

<table>
<thead>
<tr>
<th>The parameter</th>
<th>Ellipsoid</th>
<th>Krasovsky</th>
<th>Bessel</th>
<th>The differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis $a$, m</td>
<td>6 378 245.00</td>
<td>6 377 397.00</td>
<td>848</td>
<td></td>
</tr>
<tr>
<td>Semiminor axis $b$, m</td>
<td>6 356 863.02</td>
<td>6 356 078.60</td>
<td>784.4</td>
<td></td>
</tr>
<tr>
<td>Compressibility</td>
<td>0.00335233</td>
<td>0.00334280</td>
<td>0.00000953</td>
<td></td>
</tr>
</tbody>
</table>
Consequently, the geodetic heights of the points in Moscow can exceed 1000 m (Fig. 2), and the horizontal distances calculated from the coordinates may considerably differ from those measured on the city surface,

\[ \frac{\Delta S}{S} = \frac{H_r}{R} = \frac{1}{6000} \approx \frac{1}{7000} \]

where \( R \) — an average Earth's radius in Moscow.

**Results and Discussion**

The results of the influence of the geodesic height of the constructing object on the distance distortion are shown in Figure 3.

Such significant differences in the coordinates calculation results and the actual calculations are not allowed in the highly-precise geodesic measurements. It is recommended to use the subsurface of the reference in engineering and geodesic practice (Levchuk, Novak & Lebedev, 1983; Klyushyn & Mihelev, 2010), however there is no explanation what a reference surface is and how it is located on the construction site. It is only recommended to make amendments to the distance measurements results according to the formula (2) or the formula

\[ \Delta S = \frac{(H_1 - H_o) S}{R} \]

where \( H_1 \) — the height of the horizontal distance of the measured line; \( H_o \) — the height of the reference surface.

![Figure 2. The geodesic heights in Moscow](image-url)
However, this technique does not always lead to the desired results, since these recommendations do not provide the reduction of the satellite measurements results. In the engineering geodesy the other practice was widely used. After defining the position of the constructing object in the area, a local coordinate system is created at the height of the constructing object, which is not connected to the state geodesic network points. This technique is often used when constructing objects of a comparatively short time, such as bridges, dams and others, but it can not be applicable to the construction of tunnels because of their considerable length and continuous development.

The reduction of the measured quantities is usually not performed with the help of polygon measurements upon the condensation of planned reference network. But if such constructions are based on points, the coordinates of which are calculated on the reference surface, then the adjustment of such points leads to the fact that the coordinate networks of thickening would also be referred to the reference surface, as shown in Fig. 4.

Despite these rather specific features of Gauss projection, particularly this coordinate system provides the most precise and accurate maintenance of geodesic support of construction works. It is sometimes recommended to use the spherical surface as the reference surface (Klyushyn & Dinh, 2013; Gainabekov et al., 2015a; 2015b; 2015c), but with the modern computing technique it seems unnecessary to use a variety of calculation formulas.

The Gauss projection the most successfully takes into account the curvature of the Earth and has a rigorous mathematical connection with the ellipsoidal coordinates.
Moreover, the coordinate system in Gauss projection successfully meets the procedure of taking into account the curvature of the Earth, installing the tool on a vertical line with the help of a level at each determined point (Figure 5).

![Figure 5. The example of Gauss projection](image)

There are some peculiarities concerning the geodesic works when constructing the tunnels. This is due to the fact that the planned tunnel route is created on the land surface, the coordinates of the city geodesic network are assigned to the ellipsoid surface, and the tunnel is commonly built at a different height (see Fig. 6).

![Figure 6. The tunnel construction](image)

In this case it is better to take the ellipsoid coordinates at the vertical line ABC (see Fig. 5), which is the closest to the normal,

$$B_o = B_c, \quad L_o = L_c$$

... to count the geodesic heights of A and B points with the constant with an error no more than 3-5 m, and then calculate the scaling coefficient

$$k_A = \frac{1 + H_A}{R}, \quad k_B = \frac{1 + H_B}{R}$$

where $H_A$ - the average measurement of points geodesic height of planned reference network on the land surface; $H_B$ - the average geodesic height of constructing tunnel; $R$ — an average Earth’s radius;
using the data of the scaling coefficient calculate the semimajor axis $a'$ and the semiminor axis $b'$ of subordinate ellipsoids:

$$a' = ak_a; \quad b' = bk_b$$

$$a' = ak_a; \quad b' = bk_b$$

Such transformations of the main ellipsoid parameters allow to completely keep all the calculation formulas of Gauss, up to coefficients, including number values of the first and the second eccentric position, $e^2$ and $e'^2$. Using the new parameters of subordinate ellipsoids, the coordinates $x$ and $y$ shall be calculated by the Gauss formulas at the land surface and in the constructing tunnel. In this case, the ellipsoid coordinates for the points $B$, $L$ are assumed equal for all points of the vertical line on the construction site, and the coordinates in Gauss projection will be different. However, this will avoid further distortion and, the most importantly, there will be carried out the complete agreement between the coordinates and the measured values within each of the selected reference surfaces. This technique is mostly effective when using satellite measuring methods in order to create a planned tunnel route on the Earth’s land surface.

**Conclusion**

Thus, while conducting highly-precise engineering and geodesic works all the peculiarities of unique coordinate systems used in geodesy should be precisely considered. Only in such case the potentially high accuracy of modern measuring instruments could be realized. In order to conduct different engineering and geodesic works the appropriate scientifically proved methodological recommendations should be made.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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