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# A Fixed Point Theorem for Triangular Surface Mapping and Functions of Different

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**ABSTRACT**
As indicated in the example, the presence of fixed -points, Triangular Surface mapping and requirements for uniqueness can be tolerated but it is not important.

**KEYWORDS**
Fixed Point,Functions, Triangular Surface

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## Introduction

The solution of some problems in mathematics is finding a solution to an equation that can be written as h(t) = t for an appropriate h function.

We know that solution of such equations are called fixed -point and the theorems which examine the existence of these fixed points are called fixed point theorems. Fixed- point and fixed point theorems play an important role in solving the problems of existence and uniqueness in the analysis, geometry and topology, which are the subdivisions of mathematics.

Today, many researchers attempt to create different application areas by expanding the problems related to fixed point. The foundations of the fixed point theory were taken in the early 1900s with the work of L. E. J. Brouwer.

The first known fixed point theorem is that if $T: Y \rightarrow Y$ is a continuous transformation there is a fixed point in Y, as $Y = [a, b] \subset R$, T.

In 1912, Brouwer made this theorem as a continuous transformation of

$T: Y \rightarrow Y$, carrying the m-dimensional $R^m$ Euclidean space and Y $\subset R^m$ closed circle[5,6].

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In this case, he expanded so that "T function has at least one fixed point in Y." Fixed point theory evolves in two ways as shown in the studies. The fixed point theory for continuous transformations defined on the compact convex subsets of the first-order normed linear spaces and the fixed point theory for transformations similar to shrinkage and contraction defined on full metric spaces. Fixed point theory work on full metric spaces began with the Polish mathematician S. Banach in 1922. Banach gave the following theorem, also known as the principle of shrinkage transformation. [5,6]

"(Y, $d$) is a complete metric space and T: Y → Y for each, $x, y \in$ Y $d$(T$x$, T$y$) $\leq \beta d(x, y)$."

Then the transformation T has a single $\beta \in$ Y fixed -point.

**1.1. Definition** : The point p is a fixed point of the function h(x) if h(p) = p [2].

**1.2. Definition** : The point p is a root of the function h(x) if h(x) = 0. [2]

That is to say, t is a fixed point of the function T(x) if and only if T(t) = t. This implies an important terminator when calculating

T(T(...T(t)...)) = $T^m(t)$ = t

T repeatedly.

This means

T(T(...T(t)...)) = $T^m(t)$ = t

an important terminating consideration when recursively computing T. Sometimes it called fixed sets the fixed set points. For example, in the real number

y = T (x) = x is specified, 5, T is a fixed point, since T (5) = 5.

Some functions do not have fixed points: for example, if T is a function defined as

T (t) = t + 1 in real numbers, then there is no fixed -point because t is never equal to

t + 1.

In that case, there are no fixed points, because t. It is never equal to t + 1 for any real number. In terms of graphics, a fixed- point, point (T, T (t)), T (t) = t; In other words, the graph T has a common point with that line.

The point returning to the same value after the iteration of the function in the repeated number is called the periodic point. The fixed point is a periodic point and the period is an equal. A reflective geometry is called a fixed point of a projection as a double dot. (figur.1)[3].

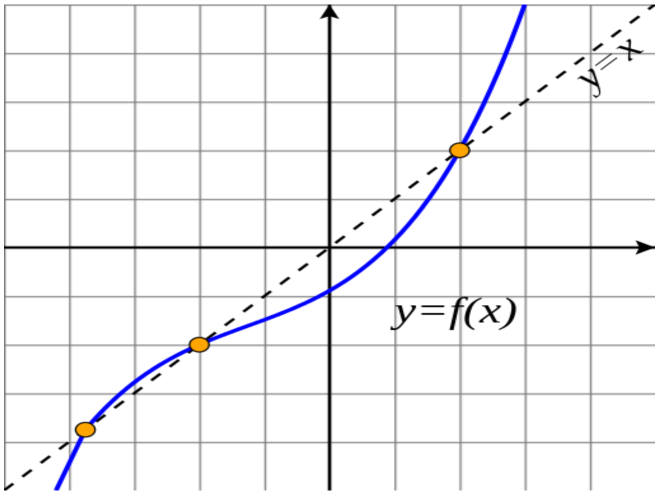


Figure1.1

**1.3. Definition** : Given a set X and a function $f : X \to X$, $x^* \in X$ is a fixed - point of f f f… f($x^*$) = $x^*$ Many existence problems in economics for example, existence of competitive equilibrium in general equilibrium theory, existence of Nash in equilibrium in game theory – can be formulated as fixed- point problems. Because of this, theorems giving sufficient conditions for existence of fixed points have played an important role in economics[8].

## Examınatıon of Mathematıcal Equatıon

We know from functional analysis lessons how to use fixed-point iteration to solve a nonlinear equation of By transforming the form Y = h (y) in to the equation for H (y) = 0

After choosing an initial guess $y^{(0)}$, we calculate a sequence of iterates by,

$y^{(m+1)} = h(y^m)$ ; m = 0, 1, 2, …,n

This, of course, it becomes a solution of the main equations.

We will learn the following: function h is a continuous mapping function interval E into itself, then h is a fixed-point $y^*$ in E, which is a point that satisfies

$y^* = h(y^*)$

That is, a solution to h(x) = 0 exists within I. Furthermore, if there is a constantq< 1 such

$|h'(y)| < q, \ y \in E,$

q fixed values may be used to specify the convergence speed,

Fixed- point repetition corresponds to Spectral radius q (T) of the iteration matrix.

$$T = S^{-1}N$$

used in a stationary iterative method of the form

$y^{(m+1)}$=T$y^m$ +$S^{-1}d$

for solving Ay = d , where A = $S^{-1}$N .

Where in the fixed point iteration. we have generalized the non-linear equation solving problems.

If the equations are unknown m, then this fixed point is unique

$h_1(y_1 y_2 \dots y_m) = 0$

$h_2(y_1 y_2 \dots y_m) = 0$

……….

$h_m(y_1 y_2 \dots y_m) = 0$

We can express this equation is a vector format to simplify the system.

H(y) = 0 and H : E⊆ $R^m \to R^m$ is a vector-valued function of the variables represented by the vectory ,

$(y_1, y_2, \dots , y_m)$ and $h_1$,$h_2$,…,$h_m$ are the component functions, or coordinate functions of H.

Boundary and continuity concepts are generalized to vector valued functions. Therefore, the functions of many variables are expressed in a simple way.

Given a function h : E ⊆ $R^m \to$ R and a point $y_0$ ∈E, we write

lim h(y) = L

$y \to y_0$

if, for any $\varepsilon$> 0, there exists a $\delta$> 0 such that

|h(y) − L |<$\varepsilon$

while y ∈ E and

0 <‖$y - y_0$‖<$\delta$

In this definition, we can get any opportune vector norm ‖ ‖.

We also have that h is continuous at

a point $y_0$ ∈E if,

lim h(y) = h($y_0$)

$y \to y_0$

If the partial derivatives of the h function are bounded by $y_0$, then it says $y_0$ is continuous.

We define the bounds for multivariable continuity and constant-valued functions.

After obtaining this we could construct this approaches for vector valued funcitons.

Given H : E⊆ $R^m \rightarrow R^m$ and $y_0 \in$ E, we saythat

lim h(y) = L

$y \rightarrow y_0$

if and only if

lim $h_j$ (y) = $L_j$, j=1,2,3,…,m

$y \rightarrow y_0$

We say that the $h_i$ coordinate functions are continuous at $y_0$and the H function is constant at $y_0$.

Equivalently, H is continuous at $y_0$ if

lim H(y) = H($y_0$)

$y \rightarrow y_0$

We can now define the fixed point iteration to solve this nonlinear system of equations.

H(y) = 0

First, let we transform this equation system to equivalent system of form

$y$= P(y)

An approach to do this is to solve the first equation for $y_j$in the original system.

Corresponds to the derivation of the Jacobi method to solve the systems of linear equations.

Then,we will select the initial guess $y^{(0)}$. The next iterations are as follows,

$y^{(m+1)}$= P($y^m$) , m = 0, 1, 2,…,n

We can define the existence of fixed points of multivariable vector-valued functions and the singularity as defined in Single Variable Functions.

The function P has a fixed point in a domain E⊆ $R^m$if P maps E into E.

If there is a constant q < 1 such that, In some natural matrix norms,

$\|J_P(y)\| \leq q$ , $y$ ∈E

JP (y) is the Jacob matrix of the partial derivatives of the P function evaluated at point y. Then P has aunique fixed point $y^*$ in E and fixed-point iteration is guaranteed to converge to$y^*$ For any chosen initial guess of E.

The inaccuracy can be seen by calculating the highly variable Taylor expansion

$$y^{(m+1)} \to y^*.$$

In the neighborhood $y^*$.

The constant of a function measures the convergence rate of the fixed-point iteration at each iteration where the q factor is reduced.

Fixed-point functions for several variables using a similar approach to the Jacobi method for linear systems it is important to obtain the Gauss-Seidel method to accelerate the convergence of the iterations. That is, when computing $y_j^{m+1}$by evaluating , we replace $y_j^m$ , for i < j, by $y_j^{m+1}$ , since ithas already been computed (assuming all components of $y^{(m+1)}$ are computed in order). Arguments of that, as in the Gauss-Seidel, we are using the most current information available when calculating every literate.For example,consider the system of equations

$$y_1 = y_2^2$$

$$y_1^2 + y_2^2 = 4$$

The first equation describes a parabola, while the second describes the unit circle. By graphing both equations, it can easily be seen that this system has two solutions, one of which lies in the first quadrant $( 2 \geq y_1 \geq 0 \; and \; 2 \geq y_2 \geq 0)$.

To solve this system using fixed-point iteration, we solve the second equation for $y_2$ and obtain the equivalent system

$$y_2 = \sqrt{4 - y_1^2} \;, y_1 = y_2^2$$

If we consider the rectangle $\mathrm{E} = \begin{Bmatrix} (y_1, y_2);\; 0 \leq y_1 \leq 2 \; and \\ 0 \leq y_2 \leq 2 \end{Bmatrix}$

we see that the function

$$\mathrm{P}(y_1 , y_2 ) = (\; y_2^2 , \sqrt{4 - y_1^2})$$

$$= (p_1 \;(y_1 , y_2 ) , p_2(y_1 , y_2 ))$$

maps E into itself. Because P is also continuous on E, it follows that P has a fixed -point in E. However, P has the Jacobian matrix

$$J_P(y) = \begin{bmatrix} 0 & 2\, y_2 \\ -y_1/\sqrt{4 - y_1^2} & 0 \end{bmatrix}$$

which cannot satisfy $\|J_P(y)\| \leq 2$ on E. Therefore, we cannot guarantee that fixed-point iteration with this choice of P will converge and in fact, as shown above it does not converge. Instead, iterations tend to approach the quadrants of E, at which they remain. In an attempt to achieve convergence, we note that

$$\frac{\partial p_2}{\partial y_2} = 2y_2 \geq 1$$

near the fixed point.Therefore, we modify P as follows:

$$P(y_1, y_2) = (y_2^2, \sqrt{4 - y_1^2})$$

For this choice of P, JPstill has partial derivatives that are greater than 4 in significantly near the fixed point. There is an important distinction here: near the fixed point, q(Jacobian P) $\leq$ 2 , whereaswith the original choice of P, q(JacobianP) $\geq$1.

Fixed- point iteration is tried and we see that with the new P, no matter how slow the convergence is, it is really satisfied.

## Result

As indicated in the example, the presence of fixed- points and requirements for uniqueness can be tolerated but it is not important.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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