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Structures of Mathematical Modeling of Metathematic and Metacognitive Skills and Abilities' Typology

Arseniy Lebedev[a], Tatsiana Krupa[a] and Maksim Rezakov[b]

[a]Laboratory of Innovations, Ltd.,Kazan, RUSSIA; [b]Astrakhan Institute of Education, Astrakhan, RUSSIA

ABSTRACT

According to the survey of experts, the association of non-cognitive skills in groups according to their classification as object of evaluation leads to the fact that one group includes very different skills - both in volume and in the way of they identify and assess. So, the purpose of the research is the development of mathematical model of metathematic and metacognitive skills and abilities' typology, as well as its subsequent extension to mathematical model of map of the present skills. As the basis for such typology codifiers developed in terms of FGES (Federal Governmental Educational Standard) requirements and corresponding systems of planned results within the framework of research on evaluation of education quality in elementary school, as well as within the framework of instrumentation for monitoring of dynamics of use in educational activity of "ICT as the instrument of activist approach realization" model by institutions of primary and general education and teachers of such educational structures was taken. The specified codifiers have been brought to unified codifier. Some objects of grading in unified codifier have been redefined in conformity with goals of the present research.

KEYWORDS
Codifier, noncognitive skills, objects of grading, model

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Introduction

The survey of teachers on GlobalLab platform revealed the consolidation of noncognitive skills in groups by their reference to object of grading (the principle taken as a basis of codifiers) (Bolbakov, 2015) is perceived by teachers-practicians as artificial (Belova, 2014). In opinion of many respondents (Kazaeva, 2016), such an approach leads to the situation (Bulgakov, 2016) when one group includes absolutely different skills - whether by the volume or by the way of their distinguishing and evaluation (Bogdanov & Khorunzhiy, 2016).

Materials and methods

CORRESPONDENCE Arseniy Lebedev ✉nakendlom@gmail.com

Within the framework of the present research the more practically grounded approach is the development of n-dimensional space of skills with subsequent clusterization of skills by any of generally-accepted methods applying computer-assisted teaching (for example, the method of k-means) (Galiaskarova, 2015). Here we should underline the fact the applying of this classification method is being purely empiric and proceeds from the premise that such a method cannot possibly relinquish to codifier method, and even in the case of total failure in matters of distinguishing of explicit clusters will provide new data for research (Zaytseva, 2015).

On the basis of above-mentioned principle we have developed 6 scales, each of which had been designed for characterization of definite noncognitive skill from the viewpoint of one of the following aspects:

accessibility of observation by a teacher or a system of automatic journalizing;

— temporal character of skill manifestation;

— entirety of a skill in the context of its fixation;

— "aspectuality" of a skill.

Each of the six measurement scales possesses five discrete marks corresponding to the degree of manifestation of this or that attribute within a definite skill. Below there are list and explanations for all the developed measurements.

Results, Discussion and Conclusions

Graduality. The attribute of graduality reflects the ability of skill to manifest with various degree of intensity. A skill with high graduality degree is perceived as learning gradually, stage by stage. An observer dealing with such a skill can say it is "developed to a greater or a lesser degree". Skills with low graduality degree can be described with binary opposition "there is a skill - no skill". It is evident that graduality is closely related to the degree of heterogeny (diversity) of actions that define it. The most striking instances of skills with low graduality are often described by the only predicate. For example, reading, writing, talking, counting - all these skills are often perceived by an observer as binary. Nevertheless, it is evident that such a skill as reading or writing is not veritably binary. Thus, for example, in respect of reading we can speak about some degrees and stages of learning: knowing of letters, spelling, reading aloud and silently. However, in the majority of cases "the ability to read" is perceived as something that is either peculiar or not peculiar to an individual. In table 1 there are explanations on scale marks for measurement of graduality.

Table 1. Degrees of graduality attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Skill is perceived as absolutely binary - it either is or not, there cannot be any interim options. An exaggerated example: the skill to add double-digit numbers. Such a skill can be easily and quickly checked by simple tests or fast observations.
0,4	Skill is perceived as close to binary. Observer can appraise its holding as incomplete. At that there are not many degrees of holding, they are strongly marked and follow one by one. Example: reading skill, in which four stages can be distinguished and all of them are clear-cut and short-termed (knowing of letters, spelling, reading aloud, silent reading).

0,6	Skill is not perceived as binary, however its learning is being strongly marked. There is an easily-distinguishing stage when observer is ready to clearly confirm a pupil holds the skill. At the same time the observer quite easily distinguish cases when the pupil has learned the skill not to the full extent, partially or insufficiently.
0,8	Skill is perceived as gradual. Observer hardly distinguishes stages of skill formation. Stages of the skill are too vague, badly detached from each other, and the observer has doubts in their presence. In addition, the observer can make conclusions about presence of skill's rudiments on the one hand, and can clearly distinguish high degree of its holding on the other hand.
1	Skill is extremely gradual. Observer often has no dead certainty at what stage of development the pupil's skill is. Transitions between stages are very vague, their order is almost unevident.

Stippling. The attribute stippling reflects temporal property of skill: in what interim it is observed, if its manifestation in time being extended or compact. A skill with high stippling degree is perceived as "quick", "compact", it can be distinguished within relatively short interim. A skill with low stippling degree manifests itself within large interim, which can often be explained by, for example, delayed character of result or large and extended in time variation of behavior corresponding to the skill. In table 2 there are explanations on scale marks for measurement of stippling.

Table 2. Degrees of stippling attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Skill is perceived as quickly manifesting in time. To distinguish it one needs just brief observation or testing of pupil. At that the skill can be rather complicated and include quite expanded variation of behavior.
0,4	Skill manifests itself in quite a fast way, however its diagnostics requires more than mere observation or rapid test. At that the first features of skill's presence are distinguished rather quickly, but there is a necessity of additional observation for confirmation of initial impressions.
0,6	Skill possesses average stippling. Its quick distinguishing (due to 1 lesson or with help of 1 test) is rather difficult, however a teacher can put together those types of activity, which will promote faster manifestation of the skill. For example, research skills manifest much faster (during days or weeks) in the course of project activities, while in case of usual class-lesson system their distinguishing requires long terms.
0,8	Manifestation of skill is very extended in time. The variation of behavior linked to the skill consists of a big number of actions performed in long-term intervals. It is being very difficult to suggest a type of activity accelerating manifestation of the skill, however it remains possible under definite circumstances.
1	Skill is exceedingly extended in time (months, school year). It is often stipulated by high degree of its aggregating. Many teachers feel uncertainty in their ability to distinguish such a skill. Presence of such a skill is often stipulated with external global factors - such as effect of environment, family situation or even genetic predisposition.

Granularity. The scale of granularity is designed for measuring degree of skill's "separatability" against the background of other skills and their complexes. According to data of survey conducted on GlobalLab platform,

teachers intuitively subdivide skills into easily separated and hardly separated that are closely related to other skills or lost against their background. It is being particularly peculiar to the skills of regulative group. It's very difficult to distinguish such skills, as far as the main objects of behaviour variations for them are often represented with other skills. Thus, for example, the skill of goal-setting is insomuch basic that almost always it appears shadowed by more particular actions and their objects. Moreover, it practically cannot be distinguished "in pure form", since any test or task aimed at its distinguishing will obligatorily include a type of activity related to behavior variation associated with more granular skill - whether it is a communication, collaboration or self-regulation skill. In table 3 there are explanations on scale marks for measurement of granularity.

Table 3. Degrees of granularity attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Skill is being well-distinguishing under observations against the background of other skills. It possesses distinct features and is easily identified by several actions comprising corresponding variation of behavior. The very behavior variation consists of clear action that often possesses superficial expression. Observer experiences no difficulties in distinguishing and identification of the skill.
0,4	Skill is quite easily distinguished against the background of other skills, however observer can run into difficulties in recognition of set of observed actions sufficient for skill identification. Some actions of behavior variations have no clear superficial manifestation or are often met in other skills.
0,6	It's rather difficult to distinguish skill against the background of other ones, however there are special conditions, upon which it can be identified unambiguously. It can also be easily distinguished in connection with other skills comprising more general aggregate skill.
0,8	Skill can hardly be distinguished as independent. The variation of behaviour linked to it possesses very weak superficial manifestation. The skill is not stipple, at the same time it is often gradual.
1	Skill is not distinguished as independent, which often makes many experts to draw a conclusion about its absence as independent. Its manifestation is very difficult to describe. Another one difficult task is the creation of tests or tasks that could diagnose it in pure form. It is often regarded as necessary condition for manifestation of some more granular skill.

Testability. The scale of testability reflects the degree of how easily skill can be distinguished and identified with help of specifically developed test or task. In other words, when measuring testability one distinguishes the degree of observers' confidence in fact the skill can be checked with help of automated or manual task. In table 4 there are explanations on scale marks for measurement of testability.

Table 4. Degrees of testability attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Skill can be tested. At that to check it one needs simple test that is often short and automated. Observers easily formulate external features of the skill and prepare tasks stipulating its manifestation.
0,4	Skill can be distinguished with help of test or task, which should be rather difficult. To make them one should apply definite efforts. The tests and

	tasks are subject to partial automation, however checking requires "hand work".
0,6	Skill can hardly be tested. Many observers agree that there is a necessity of such checking form as examination or interview to distinguish it. Preliminary diagnostics of the skill can use some partially automated tests. However, the skill is quite easily distinguished in case of rather long-term observation.
0,8	It is extremely difficult to distinguish skill even with help of examinational check or interview. It's also very difficult to develop test or task, which could diagnose the skill even partially. Many observers agree that its identification requires long-term observation of pupil under condition of educational activity.
1	Skill cannot be tested. Its identification often requires very complicated abilities and competences on the part of observer. Many teachers rely on intuition in respect of diagnostics of such skills. To distinguish them there is a necessity of long-term observation of pupil including organization of various forms of educational activity. At that non-manifestation of features of skill presence even under long-term observation often does not imply absence of skill. The bigger part of actions comprising variations of behavior of untestable skills takes place without external manifestations.

Observability. The scale of observability reflects general properties of superficial manifestation of this or that skill. Skills with low degree of observability possess extremely poor superficial manifestation, many elements of behavior variation possess no external manifestation as well and are being latent or remain invisible for observer. Skills with high degree of observability, on the contrary, possess good superficial manifestation, behavior variations corresponding to them manifest to the full extent in the form of sequence of clear-cut actions. In table 5 there are explanations on scale marks for measurement of observability.

Table 5. Degrees of observability attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Skill is well observed. Related variations of behavior include a big amount of actions with physical manifestation. Sequence of such actions is clear-cut and poorly intersect sequences related to other skills (the skill can likely be evaluated as possessing high granularity).
0,4	Observation of skill and its distinguishing are being difficult due to the fact the related variations of behavior include some number of actions, which possess no physical manifestation. However these "latent" actions are not of crucial significance for the skill. Often they are being secondary, facultative and not critical for achievement of result.
0,6	Skill is hardly distinguished by means of observation. Many actions comprising behavior variations related to it possess no external physical manifestation. At that some of these actions are critical for the skill. Despite the problems of observability such a skill can possess high or average degree of testability - there can be tests and tasks that diagnose it rather well.
0,8	It is quite difficult to distinguish skill by means of direct observation. Corresponding variations of behavior include a big number of actions possessing no external physical manifestation. It is also difficult to provide conditions for the situation when important elements of behavior variations manifest in the form of observed actions.
1	It is extremely difficult to distinguish skill by means of direct observation.

	Often the conclusion of its presence is made on the basis of indirect indicators or achievement of successful result. Such skills often possess low degree of granularity and testability as well. In many cases observer relies on his experience and intuition in matters of discussion of these skills.

Reflexivity. The scale of reflexivity reflects principal possibility of skill holder to distinguish the skill. The concept of reflexivity, as it is considered in the present research, is closely related to difference between skill and ability. In table 6 there are explanations on scale marks for measurement of reflexivity.

Table 6. Degrees of reflexivity attribute manifestation in respect of skills

Mark on the scale	Explanations
0,2	Variation of behavior is easily described by skill holder. He can describe in details own actions that have led to positive result. Activity related to the skill can be attended by commentation.
0,4	Variation of behavior related to skill can be described by pupil. However such a describing often inexact, includes indications of actions that are not linked to the skill. Pupil hardly answers the question "what have you done for…" or "how did you manage to…".
0,6	Variation of behavior related to skill is hardly realized by pupil. He experiences predicaments in describing own actions referred to the skill. Nevertheless, he can discern and describe some elements of behavior variation.
0,8	Pupil is almost not able to describe own actions led to successful result. He often describes own experience as the matter of course, something that do not require some control on his part.
1	Skill is not subject to reflexion to such extent that it's often cannot be realized by pupils at all. It is extremely difficult to describe it in terms of variation of behavior. All the actions comprising the skill are perceived as sequence of operations performed automatically or in terms of reflexes.

Falling back on the above-described conceptual model of complicated skills and abilities' typology it becomes possible to formulate its heuristic mathematical model on the basis of linear-spline approximation method.

Let us consider classical and the most wide-spread model for evaluation of the degree of holding knowledge, ability or skill based on ratio of number of successfully fulfilled tasks to general number of tasks. The model is represented in the formula (1):

$$R = \frac{\sum_{i=1}^{k} R_i}{n}. \quad (1)$$

where, R – mark; R_i – number of points obtained for execution of i-task (as a rule, the incorrect answer – 0 points, correct – 1 or more points); k – number of correctly executed tasks; n – general number of tasks (k <= n).

Afterwards the final mark is calculated by the following formula (2):

$$I = \begin{cases} 1, R \le c_1, \\ 2, c_1 < R \le c_2, \\ \dots \\ M, R > c_{M-1.} \end{cases} \quad (2)$$

I – final mark, {c1, c2, ..., cM} – landmark value vector, M – maximum possible mark (for five-mark scale M = 5).

Next we introduce the matrix O = {o_{ik}} assigning the degree of applying k-th complicated skill when executing i-th task ($0 \ge o_{ik} \le 1$). Besides we introduce the matrix V = {v_{lk}} assigning mark on l-th scale described above, every k-th complicated skill (l = {1,2 ... 6}). In such a way vector W for L of scales will assign weighted coefficients of every task, which will correct result from the viewpoint of application upon obtaining of various complicated skills and abilities. This vector can be taken by formula (3):

$$W = O \times \frac{\sum_{l=1} V_l}{L} \quad (3)$$

The number of points, which is taken by tested as a result of execution of n tasks, can be defined by formula (4):

$$y = \sum_{i=1}^{n} w_i x_i, \quad (4)$$

where xi – number of points taken by tested for execution of i-task.

On the assumption of above-stated arguments the mathematical model of complicated skills and abilities' typology is aimed at search of clusters {v_{lk}} and in such a way being a part of mathematical model of map of metathematic and metacognitive skills and abilities. Types correspond to clusters found with help of k-means++ algorithm in 6-dimensional space, which, in turn, form "landscapes" of the map

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Arseniy Alexandrovich Lebedev is a head of the Laboratory of Innovations, Ltd., Kazan, Russia. Collaborates with project GlobalLab, participates in the creation of application the crowdsourcing principles' technology of the organization of design and research activity of pupils. Laboratory of Innovations, Ltd.,Kazan, Russia.

Tatsiana Viktorovna Krupa is a project manager in the Laboratory of Innovations, Ltd., Kazan, Russia. Has a PhD in Psychology. Laboratory of Innovations, Ltd.,Kazan, Russia.

Maksim Raviljevich Rezakov has graduated from the PhD course of the Russian Academy of civil service. Now is a head of Astrakhan Institute of Education, Astrakhan, Russia.

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