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Functional Dependence for Calculation of Additional Real-power Losses in a Double-wound Supply Transformer Caused by Unbalanced Active Inductive Load in a Star Connection with an Insulated Neutral

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ABSTRACT

This article deals with the problem of calculating the additional real-power losses in double-wound supply transformers with voltage class 6 (10)/0,4 kV, caused by unbalanced active inductive load connected in a star connection with an insulated neutral. When solving the problem, authors used the theory of electric circuits, method of balanced components, field and comparative experiment, modern devices for the analysis and synthesis of electric circuits. With the research results authors obtained the functional dependence, allowing calculating the additional losses in the transformer caused by unbalanced load, which differ from the similar ones due to the use phase resistance. In order to confirm the obtained functional dependence, researchers measured current, voltage and real power for each phase of “distribution transformer – unbalanced load” module. The experiment results allowed making a conclusion that the real-power losses, calculated according to the classical formula, should be adjusted in accordance with the developed functional dependence. The application of functional dependence is possible in organizations involved in the design, replacement and upgrading of transformer substations of urban and industrial distribution networks of electric power systems in order to increase energy savings in them.

KEYWORDS
Double-wound power transformer, power distribution networks, unbalanced active inductive load, real-power losses, additional transformer losses

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Introduction

Supply transformer is one of the most important elements of each electricity mains. Power transmission over long distances from the place of production to the place of consumption in today’s networks requires not less than 5-6 times of transformation in step-up and step-down transformer (Bessonov, 2006; Naidu & Kamaraju, 2013). As the distance from the power plant grows, transformer unit output decreases, while specific consumption of materials to make transformers

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and unit output losses, as well as price for 1 kW of losses increase (Serdeshnikov et al., 2005; Weedy et al., 2012). Therefore, one of the most important tasks now is to reduce the power losses in distribution transformers with voltage class 6 (10) kV. These transformers create most of the energy losses, paid by the consumer at the highest price (Hessian Ministry of Economics, Transport, Urban and Regional Development, 206).

In recent years, the asymmetry of operating conditions demanded attention again, as evidenced by a number of scientific papers (Troitsky & Kostinsky, Durdykuliev, 2012; Dzhendubaev, 2009). As public electricity consumption in a number of energy systems has exceeded the industrial electricity consumption, this led to a breach of symmetry and balance of voltage and current systems. Therefore, the task of improving the calculation and reduction of electricity losses in power distribution networks with unbalanced load remains a topical scientific and technical power problem.

Literature Review

The paper offers an accurate calculation of the efficiency factor, and therefore, the loss of power by supply transformers (Dzhendubaev, 2009; Letschert et al., 2013). In contrast to the classical approach (Bessonov, 2006), here catalog data allows to determine the efficiency of a transformer, taking into account the effect of the nature and load rate concerning low voltage (Electric energy systems: analysis and operation, 2016; Pollock & Sullivan, 2015; Besselmann, Mester & Dujic, 2014). However, the paper is done without regard to mode's asymmetry and additional losses caused by the added to transformer substitution circuit resistances of unbalanced phase load, as well as not taking into account the output transfer from one phase to another.

The paper introduces the concept of reduced real-power losses, taking into account both the losses in a transformer and energy supply elements, with the help of economic equivalent of reactive power (loss change coefficient, C_{LC}), which characterizes real losses from a power supply to a transformer, attributable to 1 kVAr of the lost reactive power (Ministry of Economics, Transport, Urban and Regional Development, 2006). For step-down transformers 6 (10)/0,4 kV is offered in power system peak hours, C_{LC} = 0,15, during the hours of power systems low use – C_{LC} = 0.1. These losses, which are based both on the power losses in a transformer and the created in power systems elements, are recommended to be calculated according to the formula (Franklin & Franklin, 2013):

$$\Delta E_{\text{aT}} = \Delta P_{\text{x}}' \cdot T_Y + \Delta P_{\text{к}}' \cdot C_L^2 \cdot T_w,$$

where the given real-power losses of no-load and short circuit, respectively, are:

$$\Delta P_{\text{x}}' = \Delta P_{\text{x}} + C_{LC} \cdot \Delta Q_{\text{x}}; \, \Delta P_{\text{к}}' = \Delta P_{\text{к}} + C_{LC} \cdot \Delta Q_{\text{к}};$$

reactive power losses of no-load and short circuit -

$$\Delta Q_{\text{x}} = S_{\text{т.}nom} \cdot U_{\text{к}}\%/100, \, \Delta Q_{\text{к}} = S_{\text{т.nom}} \cdot I_{\text{x}}\%/100;$$

T_Y – transformer working time during a year;

C_L – transformer load coefficient;

T_{w} – transformer working time under nominal load during a year;

ΔP_{x}, $\Delta P_{\text{к}}$, $U_{\text{к}}$, I_{x} – transformer catalogue data;

$S_{\text{т.nom}}$ – nominal transformer power.

Counting real transformer no-load losses and short-circuits is expedient both in carrying out process loss calculations and in justifying the economic benefit of transformers replacement (Bonnin et al., 2013).

The increase in losses is due to the reversal of distribution transformer cores, their heating, mechanical effects of vibrations, especially in short-circuit mode, as a result of repairs (Ebrahimi et al., 2014).

The analysis of the sources, stated above, showed that a few experimental results require further investigation of unbalanced modes of supply transformers. Since the unbalanced load influences the parameters of distribution transformers, there is a need to improve the calculation of the additional power losses caused by the asymmetry of the load connected to it.

Aim of the Study

This article focuses on the definition of functional dependence for calculation of additional real-power losses in double-wound supply transformers of voltage class 6 (10) kV, due to unbalanced active inductive load in a star connection with an insulated neutral.

Research questions

What are the losses from negative-sequence currents compared to direct sequence currents?

Method

When solving the problem, authors used the theory of electric circuits, method of balanced components, field and comparative experiment, as well as modern devices for the analysis and synthesis of electric circuits.

In order to confirm the obtained functional dependence for calculation of additional real-power losses in a double-wound supply transformer caused by unbalanced active inductive load in a star connection with an insulated neutral, researchers made measurements of current, voltage and real power of each phase of “distribution transformer – unbalanced load” module.

Measurements were made using the following devices:

- K-540 measuring set, serial № 1213: nominal voltages with built-in voltmeter 15, 30, 75, 150, 300, 450, 600 V; nominal currents with built-in ammeter set 0.1, 0.25, 0.5, 1, 2.5, 5, 10, 25, 50 A; nominal real power with built in wattmeter set from 0 to 30 kW within the ranges of current and voltage measurements, stated above.

- CIRCUTOR portable power quality analyzer, series AR.5, serial № 408612036. Measuring range: current 0.05 ... 5 A, 1 A ... 200; voltage 1 ... 500 V.

All used devices have accuracy class 0.5 and a certificate of calibration.

As the research object, authors used a three-phase double-wound transformer TSZ – 2,5, with nominal power of 2.5 kW·A, the voltage on the higher voltage winding of 220 V and 127 V on the low voltage winding.

To simulate unbalanced resistance load, the researchers calculated, designed and assembled a load plant, having an active resistance. To simulate unbalanced inductive load, they calculated, designed and assembled a load plant, having an inductive resistance.

The designed plants allowed to model the operating modes for “distribution transformer – unbalanced load” module:

- active balanced load;
- active unbalanced load;
- active inductive balanced load;
- active inductive unbalanced load.

These settings also allowed to explore the operating modes of a transformer when in a star connection with an insulated neutral.

Data, Analysis, and Results

The “distribution transformer – unbalanced load” module is presented as of a system of balanced EMF sources, with $\dot{E}_A = jU$, and which can be star-connected with an insulated neutral to an unbalanced active inductive load (Figure. 1), where the complex resistance phases are:

$$\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C\,;\; \underline{Z}_A = R_A + jX_A\,;\; \underline{Z}_B = R_B + jX_B\,;\; \underline{Z}_C = R_C + jX_C\,.$$

This scheme requires determining the loss of a negative-sequence current, compared with losses from a positive-sequence current, as well as full, real, reactive power, reactive power coefficient and pulsating power.

Because of the symmetry, electromotive forces of B and C phases, respectively, equal $\dot{E}_B = jU \cdot a^2$; $\dot{E}_C = jU \cdot a$. Here $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$; $a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$ are turning unit vectors of 120° and 240° counterclockwise.

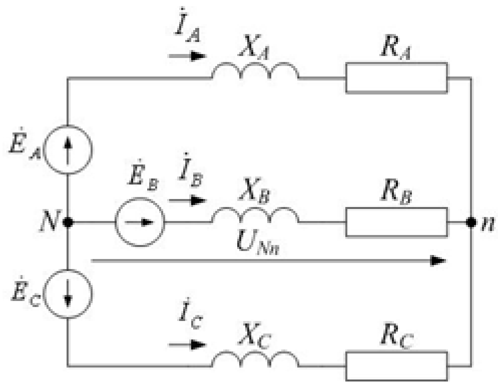

Figure 1. The three-phase network with a balanced EMF source system and an unbalanced active inductive load in a star connection with an isolated neutral.

Due to the symmetry of electromotive force source phases, we have

$\dot{E}_B = \dot{E}_A \cdot \boldsymbol{a}^2 = jU \cdot \boldsymbol{a}^2$, $\dot{E}_C = \dot{E}_A \cdot \boldsymbol{a} = jU \cdot \boldsymbol{a}$,

where $\boldsymbol{a} = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$; $\boldsymbol{a^2} = e^{j240^0} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$ are phase factors.

Load neutral offset ($\dot{U}_{Nn}$) to electromotive force neutral, according to Kennely’s formula (Bessonov, 2006) equals:

$$\dot{U}_{Nn} = jU\frac{\frac{1}{\underline{Z}_A} + \boldsymbol{a}^2 \cdot \frac{1}{\underline{Z}_B} + \boldsymbol{a} \cdot \frac{1}{\underline{Z}_C}}{\frac{1}{\underline{Z}_A} + \frac{1}{\underline{Z}_B} + \frac{1}{\underline{Z}_C}} = jU\frac{\underline{Z}_B\underline{Z}_C + \boldsymbol{a}^2 \cdot \underline{Z}_A\underline{Z}_C + \boldsymbol{a} \cdot \underline{Z}_A\underline{Z}_B}{\underline{Z}}, \quad (1)$$

where $\underline{Z} = \underline{Z}_A\underline{Z}_B + \underline{Z}_A\underline{Z}_C + \underline{Z}_B\underline{Z}_C$.

Complex phase voltages of unbalanced load with the equation (1):

$$\dot{U}_A = \dot{E}_A - \dot{U}_{Nn} = j\sqrt{3}U\frac{\underline{Z}_A\underline{Z}_B e^{-j30^0} + \underline{Z}_A\underline{Z}_C e^{j30^0}}{\underline{Z}}; \quad (2)$$

$$\dot{U}_B = \dot{E}_B - \dot{U}_{Nn} = -j\sqrt{3}U\frac{j\underline{Z}_A\underline{Z}_B + \underline{Z}_B\underline{Z}_C e^{j30^0}}{\underline{Z}}; \quad (3)$$

$$\dot{U}_C = \dot{E}_C - \dot{U}_{Nn} = j\sqrt{3}U\frac{j\underline{Z}_A\underline{Z}_C - \underline{Z}_B\underline{Z}_C e^{-j30^0}}{\underline{Z}}. \quad (4)$$

Using the equations (2) – (4), we determine the complex linear currents:

$$\dot{I}_A = \frac{\dot{U}_A}{\underline{Z}_A} = j\sqrt{3}U\frac{\underline{Z}_B e^{-j30^0} + \underline{Z}_C e^{j30^0}}{\underline{Z}}; \quad (5)$$

$$\dot{I}_B = \frac{\dot{U}_B}{\underline{Z}_B} = -j\sqrt{3}U\frac{j\underline{Z}_A + \underline{Z}_C e^{j30^0}}{\underline{Z}}; \quad (6)$$

$$\dot{I}_C = \frac{\dot{U}_C}{\underline{Z}_C} = j\sqrt{3}U\frac{j\underline{Z}_A - \underline{Z}_B e^{-j30^0}}{\underline{Z}}. \quad (7)$$

Then we apply Fortescue transformations to determine the balanced components of linear currents.

The sum of the right sides of equations (5) - (7) are equal to zero, which corresponds to the physics of the phenomenon. Since the load neutral is isolated (zero conductor resistance is infinite), then the current in the neutral conductor is $\dot{I}_N = 0$.

Complex currents of positive and negative sequences of *A* phase are respectively:

$$\dot{I}_{1A} = \frac{1}{3}\left(\dot{I}_A + \boldsymbol{a} \cdot \dot{I}_B + \boldsymbol{a}^2 \cdot \dot{I}_C\right) = jU\frac{\underline{Z}_A + \underline{Z}_B + \underline{Z}_C}{\underline{Z}}; \quad (8)$$

$$\dot{I}_{2A} = \frac{1}{3}\left(\dot{I}_A + \boldsymbol{a}^2 \cdot \dot{I}_B + \boldsymbol{a} \cdot \dot{I}_C\right) = -jU\frac{\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C}{\underline{Z}}. \quad (9)$$

By adding the right sides of equations (8) and (9) we obtain an expression similar to the right-hand side of equation (5), which is a credibility test for the transformations of positive and negative currents of *A* phase sequence, i.e.

$$\dot{I}_A = \dot{I}_{1A} + \dot{I}_{2A}.$$

Balanced components of *B* and *C* phases, respectively, may be determined by using for this purpose the phase factors:

$$\dot{I}_{1B} = \dot{I}_{1A} \cdot \boldsymbol{a}^2; \ \dot{I}_{2B} = \dot{I}_{2A} \cdot \boldsymbol{a};$$

$$\dot{I}_{1C} = \dot{I}_{1A} \cdot \boldsymbol{a}; \ \dot{I}_{2C} = \dot{I}_{2A} \cdot \boldsymbol{a}^2.$$

Formulas (1), (5 - 7) derive from the expanded matrix equations (Danko, 2005; Almeida & Kato, 2014) of unbalanced three-phase active inductive load mode in a star connection with an insulated neutral:

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ \underline{Z}_A & 0 & 0 & 1 & \dot{E}_A \\ 0 & \underline{Z}_B & 0 & 1 & \dot{E}_B \\ 0 & 0 & \underline{Z}_C & 1 & \dot{E}_C \end{array}\right).$$

Asymmetry coefficient modulus currents in negative current:

$$K_2 = \left|\dot{K}_2\right| = \left|\frac{\dot{I}_{2A}}{\dot{I}_{1A}}\right|.$$

Additional active power losses in relative phase units are equal to the square current asymmetry coefficient modulus of negative current (Troitsky, 2001):

$$\Delta P^* = \left|\dot{K}_2\right|^2 = \left|\frac{\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C}{\underline{Z}_A + \underline{Z}_B + \underline{Z}_C}\right|^2. \quad (10)$$

Complex expression in parentheses of equation numerator (10) is

$$\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C =$$
$$= R_A - \frac{R_B + R_C}{2} + \frac{\sqrt{3}}{2}(X_C - X_B) + j\left(X_A - \frac{X_B + X_C}{2} + \frac{\sqrt{3}}{2}(R_B - R_C)\right). \quad (11)$$

Square modulus of the complex expression in the equation (10) denominator:

$$\left|\underline{Z}_A + \underline{Z}_B + \underline{Z}_C\right|^2 = Z_A^2 + Z_B^2 + Z_C^2 + 2(R_A R_B + X_A X_B) + 2(R_A R_C + X_A X_C) +$$
$$+2(R_B R_C + X_B X_C). \quad (12)$$

Square modulus of the complex expression (11):

$$\left|\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C\right|^2 = Z_A^2 + Z_B^2 + Z_C^2 - R_A R_B - R_A R_C - R_B R_C - X_A X_B -$$
$$-X_A X_C - X_B X_C + \sqrt{3} R_A (X_C - X_B) + \sqrt{3} R_B (X_A - X_C) + \sqrt{3} R_C (X_B - X_A). \quad (13)$$

Insymboling the following expressions:

$$\alpha = \left|\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C\right|^2; \ \beta = \left|\underline{Z}_A + \underline{Z}_B + \underline{Z}_C\right|^2.$$

Substituting the values on the right sides of equations (12) and (13), based on the received symbols in the formula (10), we obtain an expression of additional active power losses in p.u. in the case of unbalanced active inductive three-phase load in a star connection with an insulated neutral, with the restrictions mentioned earlier in the formulation of the problem:

$$\Delta P^* = \frac{\alpha}{\beta}. \quad (14)$$

If inductive resistance load phases are equal and active are not, i.e. $X_A = X_B = X_C = X;\ R_A \neq R_B \neq R_C$, then, according to (14):

$$\Delta P_1^* = \frac{R_A^2 + R_B^2 + R_C^2 - R_A R_B - R_A R_C - R_B R_C}{R_A^2 + R_B^2 + R_C^2 + 2R_A R_B + 2R_A R_C + 2R_B R_C + 9X^2}. \quad (15)$$

When reactive phase load is also compensated ($X = 0$), then

$$\Delta P_2^* = \frac{R_A^2 + R_B^2 + R_C^2 - R_A R_B - R_A R_C - R_B R_C}{(R_A + R_B + R_C)^2}. \quad (16)$$

Having $R_A = 1, r_b = \frac{R_B}{R_A}, r_c = \frac{R_C}{R_A}$, (16) gives a similar result to the one, stated in (Troitsky & Kostinsky, Durdykuliev, 2012):

$$\Delta P_2^* = \frac{1 + r_b^2 + r_c^2 - r_b - r_c - r_b r_c}{(1 + r_b + r_c)^2}. \quad (17)$$

Figure 2 demonstrates a function graph (17) in the variation interval r_b and r_c from 0 to 1, in increments of 0,01.

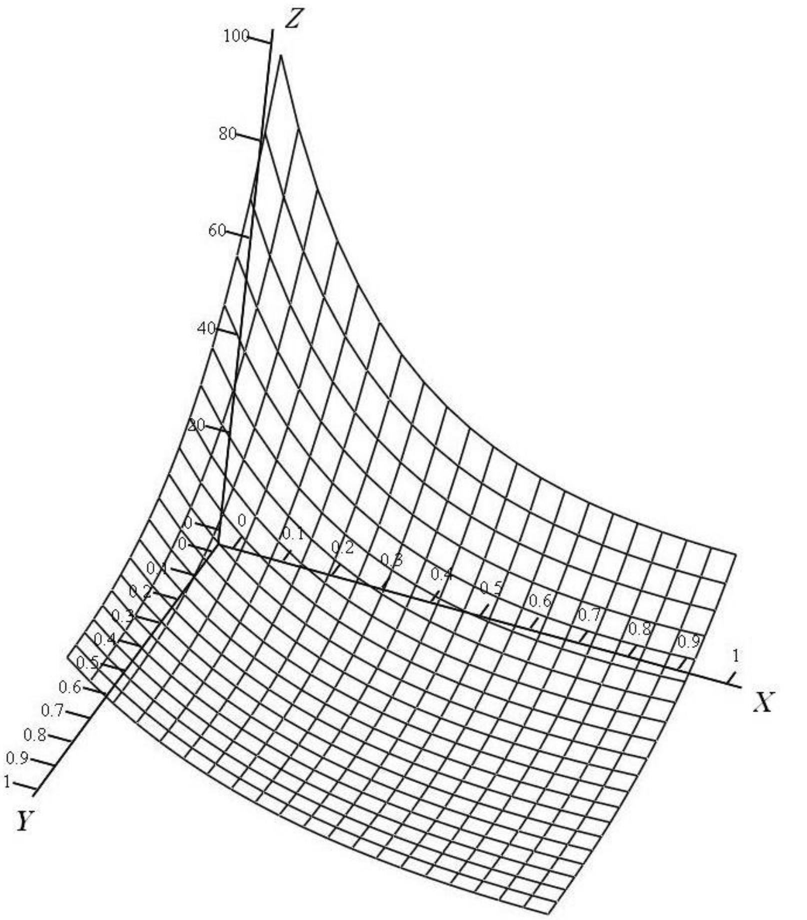

Figure 2. Function (17) graph –surfaces of the second order in the axes: X - r_b (o.e), Y - r_c (o.e), Z - ΔP_2^* (%)

In addition to $X_A = X_B = X_C = X$, we take up $R_A \neq R_B = R_C$, i.e. only one active load differs from the active phase of the other two phases of the load, from the formula (15) we have :

$$\Delta P_1^* = \frac{(R_A - R_B)^2}{(R_A + 2R_B)^2 + 9X^2}; \ \Delta P_2^* = \frac{(R_A - R_B)^2}{(R_A + 2R_B)^2},$$

And from formula (17) –

$$\Delta P_2^* = \frac{(1 - r_b)^2}{(1 + 2r_b)^2}. \quad (18)$$

Suppose $R_A = n \cdot R_B$, then:

$$\Delta P_2^* = \frac{(n-1)^2}{(n+2)^2}. \quad (19)$$

Having equal active ($R_A = R_B = R_C = R$) and unequal inductive ($X_A \neq X_B \neq X_C$) load phase resistances formula (14) transforms into:

$$\Delta P_1^* = \frac{X_A^2 + X_B^2 + X_C^2 - X_A X_B - X_A X_C - X_B X_C}{X_A^2 + X_B^2 + X_C^2 + 2(X_A X_B + X_A X_C + X_B X_C) + 9R^2}. \qquad (20)$$

If the load resistance in all the phases are the same, then $\Delta P_1^* = 0$.

When the balanced electromotive forces system is connected only to an inductive load only in a star connection with an isolated neutral, we get the formula for ΔP_2^*, similar to the formula (16), with the only difference being that instead of the active it has inductive phase resistances:

$$\Delta P_2^* = \frac{X_A^2 + X_B^2 + X_C^2 - X_A X_B - X_A X_C - X_B X_C}{(X_A + X_B + X_C)^2}. \qquad (21)$$

Suppose $X_A = 1, x_b = \frac{X_B}{X_A}, x_c = \frac{X_C}{X_A}$. In this case (21) is similar to (17), i.e.

$$\Delta P_2^* = \frac{1 + x_b^2 + x_c^2 - x_b - x_c - x_b x_c}{(1 + x_b + x_c)^2}. \qquad (22)$$

When the reactive load only in one phase, for example A, is different from reactive loads in other two phases, then from equations (20) and (21) it follows that

$$\Delta P_1^* = \frac{(X_A - X_B)^2}{(X_A + 2X_B)^2 + 9R^2}, \Delta P_2^* = \frac{(X_A - X_B)^2}{(X_A + 2X_B)^2},$$

and from formula (22) –

$$\Delta P_2^* = \frac{(1 - x_b)^2}{(1 + 2x_b)^2}. \qquad (23)$$

Supposing $X_A = n \cdot X_B$, we have (19).

If the load is only one phase, for example A, i.e. $R_A + jX_A \neq 0, \underline{Z}_B = 0, \underline{Z}_C = 0$, then (see (14)):

$$\Delta P^* = \frac{Z_A^2}{Z_A^2} = 1.$$

This particular case confirms the “capability” of the general formula for the additional losses from the negative-sequence currents.

Full power of the considered unbalanced load is the sum of the power phases:

$$\dot{S} = \dot{S}_A + \dot{S}_B + \dot{S}_C = \frac{\dot{U}_A^2}{\underline{Z}_A^2} \cdot \underline{Z}_A + \frac{\dot{U}_B^2}{\underline{Z}_B^2} \cdot \underline{Z}_B + \frac{\dot{U}_C^2}{\underline{Z}_C^2} \cdot \underline{Z}_C.$$

Using the complex values of the phase voltages of (2) - (4), we define the values of the square moduli.

Square complex phase voltage A:

$$\dot{U}_A^2 = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_A|^2 \cdot |\underline{Z}_B e^{-j60^0} + \underline{Z}_C|^2.$$

Total phase A capacity:

$$\dot{S}_A = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_B e^{-j60^0} + \underline{Z}_C|^2 \cdot \underline{Z}_A.$$

Squares of complex voltages and total capacity phases B and C:

$$\dot{U}_B^2 = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_B|^2 \cdot |\underline{Z}_A e^{j240^0} - \underline{Z}_C|^2;\ \dot{S}_B = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_A e^{j240^0} - \underline{Z}_C|^2 \cdot \underline{Z}_B;$$

$$\dot{U}_C^2 = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_C|^2 \cdot |\underline{Z}_A e^{j120^0} - \underline{Z}_B|^2;\ \dot{S}_C = \frac{3U^2}{|\underline{Z}|^2} \cdot |\underline{Z}_A e^{j120^0} - \underline{Z}_B|^2 \cdot \underline{Z}_C.$$

Total load capacity:

$$\dot{S} = \frac{3U^2}{|\underline{Z}|^2} \cdot \Big(\underline{Z}_A |\underline{Z}_B e^{-j60^0} + \underline{Z}_C|^2 + \underline{Z}_B |\underline{Z}_A e^{j120^0} + \underline{Z}_C|^2 + \underline{Z}_C |\underline{Z}_A e^{j120^0} - \underline{Z}_B|^2\Big). \quad (24)$$

Formula (24) can be written in a compact form:

$$\dot{S} = \frac{3U^2}{Z^2} \cdot \left(d \cdot \underline{Z}_A + e \cdot \underline{Z}_B + f \cdot \underline{Z}_C\right), \quad (25)$$

Where:

$$d = |\underline{Z}_B e^{-j60^0} + \underline{Z}_C|^2 = Z_B^2 + Z_C^2 + R_B R_C + X_B X_C + \sqrt{3}(R_C X_B - R_B X_C);$$

$$e = |\underline{Z}_A e^{j120^0} + \underline{Z}_C|^2 = Z_A^2 + Z_C^2 + R_A R_C + X_A X_C + \sqrt{3}(R_A X_C - R_C X_A);$$

$$f = |\underline{Z}_A e^{j120^0} - \underline{Z}_B|^2 = Z_A^2 + Z_B^2 + R_A R_B + X_A X_B + \sqrt{3}(R_B X_A - R_A X_B);$$

$$Z^2 = Z_A^2 Z_B^2 + Z_A^2 Z_C^2 + Z_B^2 Z_C^2 + 2Z_A^2(R_B R_C + X_B X_C) + 2Z_B^2(R_A R_C + X_A X_C) + +2Z_C^2(R_A R_B + X_A X_B),$$

and its orthogonal components – real and reactive power – are respectively:

$$P = \frac{3U^2}{Z^2} \cdot (d \cdot R_A + e \cdot R_B + f \cdot R_C); \quad (26)$$

$$Q = \frac{3U^2}{Z^2} \cdot (d \cdot X_A + e \cdot X_B + f \cdot X_C). \quad (27)$$

Using equations (26) and (27), we write the expression of reactive power coefficient for the general case of unbalanced active inductive three-phase load:

$$\tan\varphi = \frac{Q}{P} = \frac{d \cdot X_A + e \cdot X_B + f \cdot X_C}{d \cdot R_A + e \cdot R_B + f \cdot R_C}. \quad (28)$$

Functions (14), (25) ... (28) in expanded form, expressed through six arguments (RA, RB, RC, XA, XC, XB), are lengthy. Network might use them numerically. Finding their global extrema is a difficult task. Therefore, the study was carried out for the special cases that occur in the production practice.

It is known that pulsating power of a three-phase unbalanced system is equal to the sum of pulsating power phases:

$$\dot{N} = \dot{E}_A \cdot \dot{I}_A + \dot{E}_B \cdot \dot{I}_B + \dot{E}_C \cdot \dot{I}_C. \quad (29)$$

Substitute in (29) the value of the electromotive force and the current phase of the expressions (2) ... (7). After some transformations we obtain:

$$\dot{N} = 3U^2 \cdot \frac{\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C}{\underline{Z}_A \underline{Z}_B + \underline{Z}_A \underline{Z}_C + \underline{Z}_B \underline{Z}_C}. \quad (30)$$

As it follows from (30), $N = 0$, if

$$\underline{Z}_A + \boldsymbol{a} \cdot \underline{Z}_B + \boldsymbol{a}^2 \cdot \underline{Z}_C = 0,$$

which is equal to $\underline{Z}_A = \underline{Z}_B = \underline{Z}_C$.

In order to confirm the obtained functional connections for calculation of additional real-power losses in the double-wound supply transformer caused by an unbalanced active inductive load in a star connection with an insulated neutral, researchers conducted a series of experiments on the "distribution transformer - unbalanced load" model.

Based on measurements and calculations, authors built the functional connections to the real-power losses and calculation tolerances of real-power losses of the load factor. Below is an analysis of the experimental data for the star load connection scheme with an insulated neutral (Figure 1). The experimental functional connections are well approximated by polynomials of 5th degree (Figures 3 – 8).

When changing the load coefficient in the range of 0.2 - 0.4, the loss difference in unbalanced and balanced modes varies slightly, which is consistent with the physics of the process (Figure 3). We deal with no-load mode (relatively constant losses). In the range of 0.4 - 1.0 loss difference with unbalanced and balanced modes increases. Its average value 14.33%.

Using the classical formula ($\Delta P = \Delta P_{x} + C_L \cdot \Delta P_{к}$) lowers the losses, as opposed to the actual and proposed functional connections for calculating the real-power losses from unbalanced mode of active inductive load and have the smallest tolerance (Figure 4).

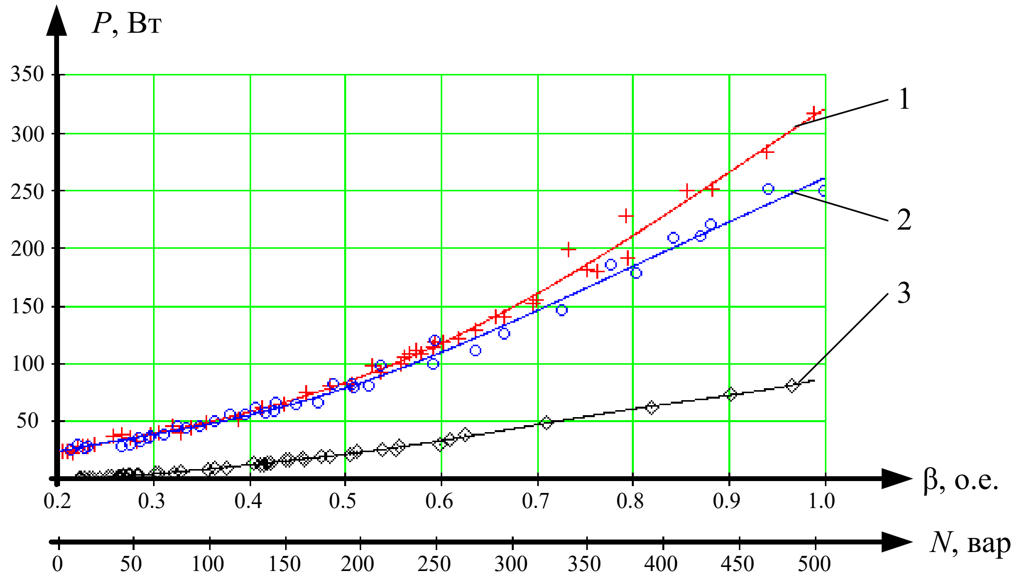

Figure 3. Functional connection of real-power losses in a transformer to the load coefficient: 1 - at unbalanced active inductive load; 2 - at balanced active inductive load; 3 - the functional connection of real power losses in a transformer at active inductive load to the pulsating power of the three phases.

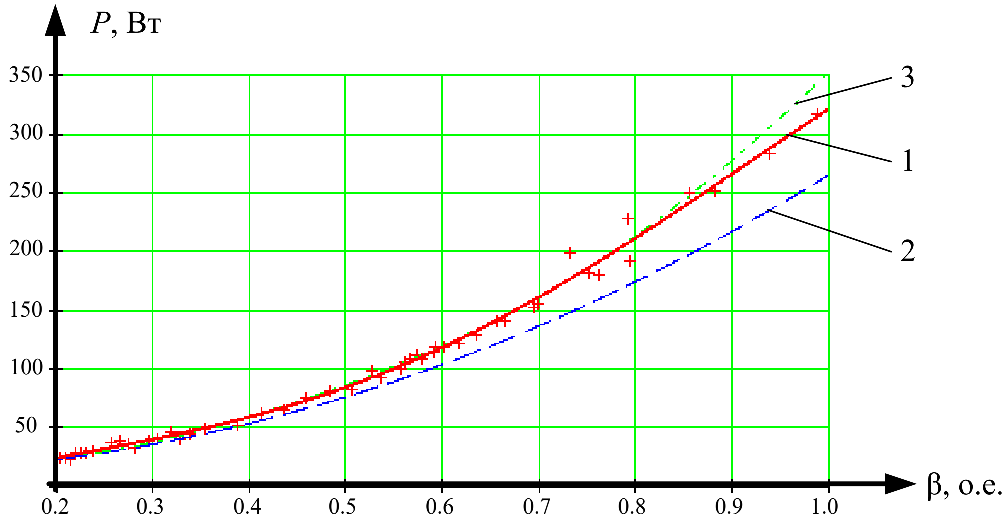

Figure 4. Functional connection of real-power losses in a transformer at the unbalanced active inductive load to the load coefficient: 1 – experimental; 2 – calculated with the classical formula; 3 – based on the asymmetry of active inductive load to the pulsating power of the three phases.

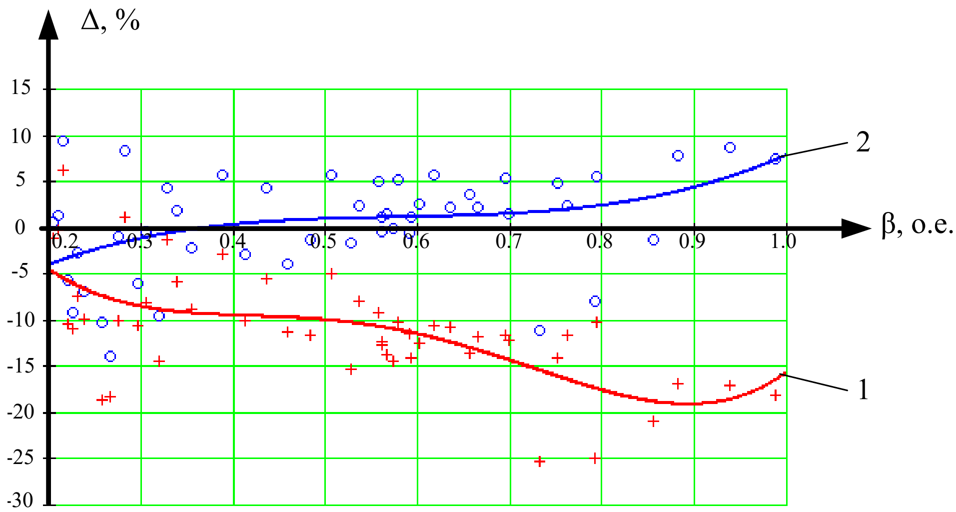

Figure 5. Functional connection of real-power losses to the load coefficient at unbalanced active inductive load: 1 – according to the classical formula; 2 – at classical formula, based on the obtained functional connection.

In the variation range of the load coefficient of 0.2 - 0.367, calculation tolerance according to the classical formula, based on the resulting functional connection, is negative, and from 0.367 to 1.0 it is positive, with a maximum tolerance of 7.98% at a load coefficient of 1.0 (Figure 5). The average tolerance of 0.2 to 1.0, according to the classical formula with load coefficient equals 11.5%, while based on the proposed functional connections of 1.34%.

The graph of real power from the pulsating power loss (Figures 3 – 6, curve 3) repeats the functional connection graph of the real-power losses to the load coefficient at unbalanced active loads, i.e. it is a feature of an unbalanced mode.

When changing the load coefficient in the range of 0.2 - 1.0, loss difference at unbalanced and balanced modes remains conditionally unchanged, as shown in Figure 6. In the range of 0.2 - 1.0 loss difference with unbalanced and balanced load is 9.62%.

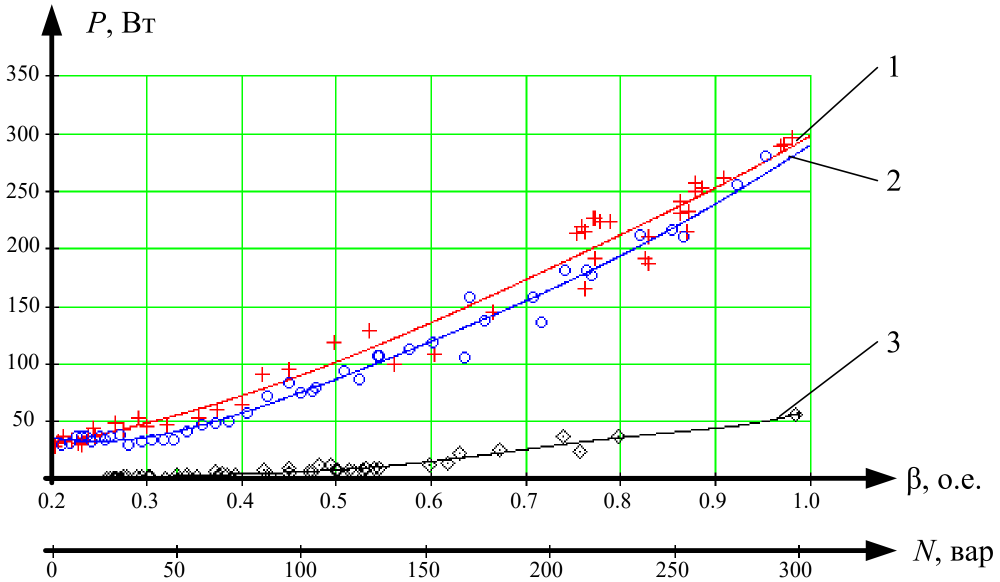

Figure 6. Functional connection of real-power losses in a transformer to the load coefficient: 1 - at unbalanced active load; 2 - at balanced active load; 3 - the functional connection of real power losses in a transformer at unbalanced active load to the pulsating power of the three phases.

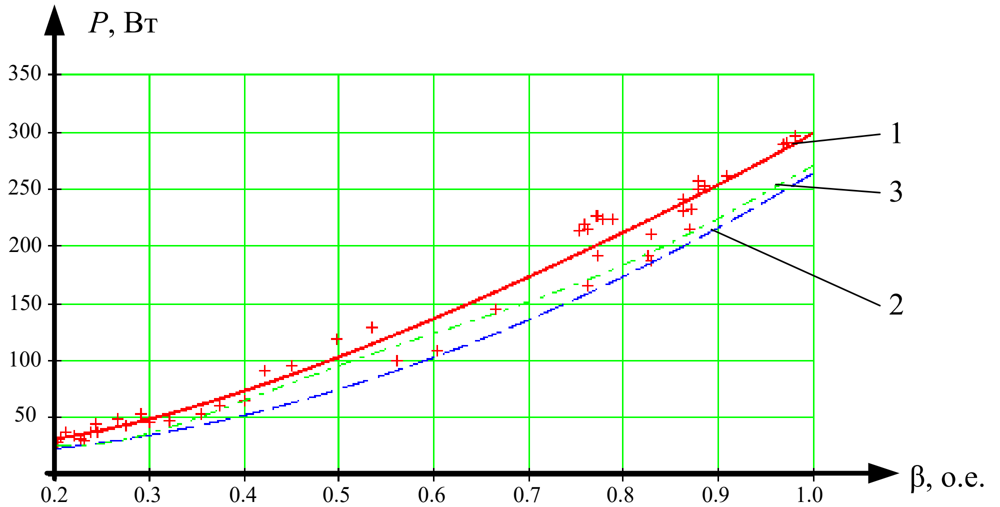

Figure 7. Functional connection of real-power losses in a transformer at unbalanced active load to the load coefficient: 1 - experimental; 2 - calculated with the classical formula; 3 - based on the asymmetry of active inductive load (using the obtained functional connection).

Calculations according to the classical formula give low loss, as opposed to the actual, while the proposed functional connection for calculating the real-power losses to active load unbalanced mode have the smallest tolerance (Figure 7).

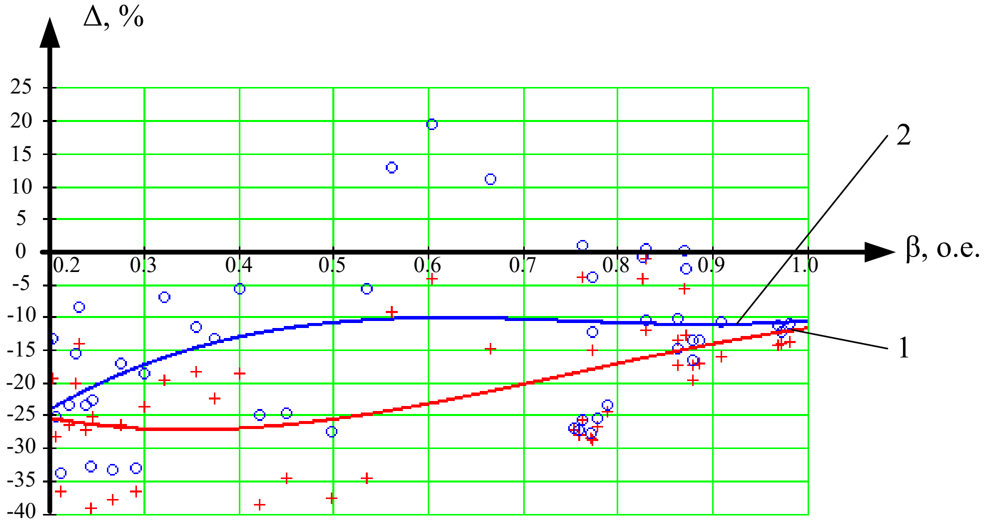

Figure 8. Functional connection of calculation tolerances of real-power losses to the load coefficient at unbalanced active inductive load: 1 - according to the classical formula; 2 - at classical formula, based on the obtained functional connection.

In the variation range of the load coefficient of 0.2 - 1.0, calculation tolerance, based on the resulting functional connection, is negative, with a maximum tolerance of -23.95% while load coefficient is 0.2 (Figure 7). The average tolerance in load coefficient of 0.2 to 1.0 according to the classical formula is -19.46%, and based on the proposed functional connection -11.33%.

Thus, the real-power losses, calculated according to the classical formula, should be adjusted in accordance with the developed functional connection.

Discussion and Conclusion

The results can be applied in the development of new technological solutions, based on the functional connection submitted in the article, aimed at improving energy efficiency and reducing active power losses in transformers of urban and industrial distribution networks of electric power systems. Using these solutions can increase the profit of companies engaged in the distribution of electrical energy, and the release of additional funds for their modernization and technical re-equipment.

From the comparison of the right sides of (15) and (16) it follows that at the compensated reactive load of phases relative values of the real-power of the negative-sequence current loss compared to the positive-sequence current loss is greater than at the uncompensated, i.e. $\Delta P_2^* > \Delta P_1^*$

Although the functions (14) and (25) … (28) in the expanded form, expressed through six arguments are lengthy and the determination of their global extrema is a difficult task, for a particular network their study is possible through numerical methods (Troitsky & Kostinsky, Durdykuliev, 2012). Due to their simplification they are convenient for programming.

Implications and Recommendations

Authors offer a functional connection to determine the value of additional real-power losses in transformers to unbalance load in a star connection with an insulated neutral. The practical value of the proposed functional connection is that it allows determining the estimated value of real-power losses in transformers from the asymmetry of the measured values of voltage, current and real power for each phase.

The application of the developed functional connection, based on the "distribution transformer – unbalanced active inductive load" module, is possible in the organizations involved in the design, replacement and upgrading of transformer substations of urban and industrial distribution networks of electric power systems in order to increase energy savings in them. The experimental data obtained in the measurement of the "distribution transformer - unbalanced load" module reaffirmed the need to adjust the classical formula for the calculation of losses in the transformer.

Disclosure statement

No potential conflict of interest was reported by the authors.

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