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Development of Intellectual Activity in Solving Exponential Inequalities

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ABSTRACT

The article describes the possibilities and the main directions of development of intellectual activity in teaching mathematics in school. The aims and specific features of application of international comparative TIMSS and PISA studies, as well as their results in the field of mathematics education in relation to the pupils in Kazakhstan are analyzed. We present some examples of solving exponential inequalities by the inverse operation method: in solving them it is possible to develop creativity and form the thinking logic of pupils.

KEYWORDS
mathematics education, intellectual activity, TIMSS, PISA, mathematical literacy, methods of inverse operations, solution of exponential inequalities, the standard inequality.

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Introduction

Mathematics education is one of the most important factors determining the level of economic and socio-political development of the society as a whole and each individual in particular.

There is a common point of view (originating from mathematicians, methodologists and pedagogues) that intellectual activity develops spontaneously in the process of learning mathematics (especially geometry) in school. Even Plato considered it necessary to include mathematics into the general education system. He claimed: "Mathematics awakens the mind, gives it flexibility, liveliness and comprehension. The main objective of teaching mathematics is not to provide a set of useful knowledge, but to create a 'clear head', so that the mind becomes capable of accommodating intelligible truth". "Who does not know mathematics, – echoes the philosopher Bacon, – cannot learn any other science, and even will not be able to discover his/her ignorance". No less known is a classic maxim by M.V. Lomonosov: "Mathematics has to be studied just because it puts mind in order".

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The study of mathematics requires constant effort of attention, ability to concentrate (a problem of today's schoolchildren); it requires persistence and reinforces the work skills. "Mathematics as a subject in school is, perhaps, the most difficult thing in education, – wrote a well-known mathematician Yu. P. Solovyov. – Mathematics in a broad sense, mathematics for all is an art of correct logical thinking, mastering spatial forms, making plausible estimates". Therefore, "mathematics, – he notes, – affects both the development of intelligence and the formation of character" (Filippov, 2000).

School is one of the social institutions created by society for upbringing the younger generations. Training is the primary means in school. The study of mathematics in this process is an important link: contributing to solving many problems of upbringing, it develops, in particular, the intellectual activity that involves:

a) mastering general methods of reasoning, proofs and solutions;

b) understanding that mathematics enables to discover regularities and express them in the most concise and convenient form;

c) understanding that mathematics “works” in various spheres of human activity;

d) certain volume of knowledge and skills of mathematical character (Reshotova, 2002).

In our opinion, there are five main directions of development of intellectual activity in the mathematics training: the subject, utilitarian-applied, psycho-pedagogical, philosophical-cultural and cognitive-aesthetic ones.

The subject direction is a kind of "game of mathematics", which is excellently described in an essay by P. Lockhart "A Mathematician’s Lament" (Lockhart, 2014). It goes under the slogan "mathematics for mathematics", because it is a self-sufficient phenomenon. In addition, this game, being an intellectual one, has its objects (mathematical concepts), rules (theorems, properties), procedures (proofs and refutations), stratagems (methods and heuristics), as well as its language, symbols and even its history, in which its revolutions took place and its heroes, outstanding mathematicians, were born.

The utilitarian-applied direction of mathematics as a science is determined by its role in the life of society and the individual; it involves the realization of intra- and inter-subject relationship of mathematics: the ability to solve everyday life tasks (the computing aspect), the use of statistical data, the ability to use the mathematical apparatus in various fields of science. Therefore, success in a wide variety of areas depends on how well the connection is mastered of the professional field with the mathematical apparatus. Mathematics searches for economical and, at the same time, substantiated solutions in any field, while helping to translate specific problems into the language of computers.

The psycho-pedagogical direction in the study of mathematics is the development of thinking and mental activity. According to V.A. Uspensky (2010), the purpose of teaching mathematics is the expansion of a pupil’s psychology, fostering the discipline of thinking: the ability to distinguish the reliable from the unreliable, the meaningful from nonsense, the understandable from the uncomprehended.

Mathematical problems help in mastering laws and properties, as well as improving the process of thinking. The foundation of the thinking process is mathematical expressions. Without obtaining information about the expression, we cannot think, neither can we solve the problem (Alpyssov, 2013; Esmukhan, & Alpyssov 2002; Alpyssov, 2012). In solving the problem, the information within the expression sets the thinking activity in motion. Mathematics is an abstract science, so without learning to think abstractly, pupils cannot form the mathematical abilities.

The philosophical-cultural direction of mathematics contributes to the formation of not only the intellectual activity of pupils, but also their world-view through a variety of concepts and the terms reflecting them. According to A.L. Zhokhov (2007), it was precisely under the influence of the natural sciences (and, in particular, mathematics) that the elements of scientific-philosophical potential were originated. Mathematics reveals the world to us through the prism of specific knowledge: the system of mathematical concepts (the infinite, the impossible, length, measure, true and false, etc.), the chains of logical derivations, introducing to the language of formulas and notations, the graphical models. All this allows forming heuristic ideas about the laws of the surrounding world.

According to the famous educator and innovator R.G. Khazankin, the cognitive-aesthetic direction of the mental activity development in learning mathematics nurtures the "sense of beauty". According to him, mathematics has its own beauty, which is in the search process (Khazankin, 1990). So, one may continually solve the same type of exercises, without thinking about the subtleties of the solution, whereas it is possible, having solved the problem in a non-standard way, to experience great pleasure.

So, the authors have listed five directions in teaching mathematics: the subject, utilitarian-applied, psycho-pedagogical, philosophical-cultural and cognitive-aesthetic ones. At the same time, the competence-based approach actualizes the utilitarian-applied direction. However, the considerations of usefulness and applicability of mathematical knowledge should not push aside the need for teaching the methods of logical deduction and proof. The activity approach primarily considers the psycho-pedagogical direction. Apparently, the philosophical-cultural direction should prevail from the positions of the culture-conforming training and the axiological approach.

Methods

Many familiar to us concepts and ideas are changing rapidly in today's world. The issue of the quality of school mathematics education is no exception. In many countries, including Kazakhstan, the ratings in the international comparative studies such as TIMSS, PISA and others are considered to be among its essential indicators. In this regard, we will consider the TIMSS and PISA indicators in the studies of the mathematics education quality.

Consider the test results for each of these studies.

The main objective of the TIMSS study is to obtain data on the state and dynamics of mathematical training of schoolchildren from various countries of the world. The scale of educational achievements, which is used in the TIMSS and PISA studies, is 1000-points, and the average result for all participating countries is taken to be 500 points.

Taking part in the TIMSS study, countries receive not only an independent international evaluation by the experts in assessing the education quality. A four-year cycle (after 4 years the fourth-graders become the eighth graders) allows tracing the trajectory of changes in the mathematics and science education in the transition from primary to basic school.

According to the results of participation of the fourth-grade pupils in the country-independent monitoring of the mathematical education quality, the scores of 27 countries are statistically higher than the average international score. In the overall standing, the success indicator of solving the test problems of TIMSS-2011 by Kazakhstan's fourth-graders was 501 points (27th place).

According to the national report data (The Results of the International Study Evaluating Educational Achievements of the Pupils of the 4th and 8th Grades of Secondary Schools of Kazakhstan. National Report, 2013), there was indicated a trend of decreasing the results of Kazakhstan's fourth-graders by 48 points compared to the previous study of 2007.

The results of Kazakhstan's schoolchildren of the 8th grade (487 points, 17th place), who took part in TIMSS for the first time, were comparable with the results of pupils from the countries such as New Zealand and Sweden. The highest results were demonstrated by the pupils of the Republic of Korea, Singapore and Chinese Taipei, Hong Kong and Japan.

The mathematical training of pupils in the TIMSS study is interpreted, in particular, in the context of a 4-level assessment system: the advanced, high, medium and low levels.

A high level of mathematical training was demonstrated by 29% of the 4-th grade pupils from Kazakhstan, which is comparable to the international average score and the results of Romania and Italy. The 4-th grade pupils from Kazakhstan more successfully perform mathematical tasks of the average level of difficulty. More than half of the 4-th grade pupils demonstrated the use of basic knowledge in solving simple problems. There was a significant reduction of the level of mathematical training in comparison with the previous high TIMSS scores (2007). Only 7% of pupils of 4th grades are at the advanced level of mathematical training. Kazakhstan's fourth-graders showed the result of one level higher than the average international score and took the 27th place in the ranking of 50 countries.

A high level of mathematical training is registered for 23% of 8th graders. Kazakhstan's eighth graders showed an average level of mathematical preparedness and took 17th place in the ranking of 42 countries.

On the basis of the TIMSS results, the following can be noted. Firstly, 33% of the international test tasks do not conform to the math curriculum in the elementary school of Kazakhstan, for the eighth grade, 5%. The content of school education in Kazakhstan is characterized by excessive "theorizing", whereas the best results were shown by the countries, where new educational programs and integrated courses have been introduced.

The main objective of the PISA study is obtaining information about the readiness of 15-year-old adolescents to adapt in modern society, including the level of mathematical literacy, that is, the readiness to use mathematics for solving problems in everyday life.

Mathematical literacy is the ability of an individual to formulate, apply and interpret mathematics in various contexts. It includes mathematical reasoning, the use of mathematical concepts, procedures, facts and tools for describing, explaining and predicting phenomena. It helps people understand the role of mathematics in the world, express well-substantiated judgments and make decisions, which should be made by the constructive, active and thinking citizen (OECD, 2013).

In PISA, the evaluation of mathematics education measured the pupils' ability to formulate, use and interpret information in different contexts and subject areas. The evaluation of mathematical literacy according to PISA included: mathematical logic; application of mathematical concepts, procedures and facts; the use of means to describe, explain and predict phenomena; the role of mathematics in the world, and the necessity to make informed decisions and express judgments required for productive, active and thinking citizens.

The system of the PISA test math assignments assesses the pupils' skills in four content-related areas:

1. "Space and form" (geometry) concerns the spatial and planar geometric shapes and their properties, it presents the tasks on recognition of shapes in different representations and sizes, determination of similarities and differences in form, as well as understanding of the properties of objects;

2. "Changes and relationships" (algebra) provided the tasks on the mathematical description of dependencies between variables in different processes, including the concept of equations, inequalities, equivalence and divisibility;

3. "Uncertainty and data" (the probability theory and statistics) includes the questions on probability and statistics;

4. "Quantity" (arithmetic) contains the tasks on quantitative relations and regularities, including quantitative representations, the concepts of area and volume, mental calculation, approximate calculation and understanding the meaning of mathematical operations.

According to the available data (The World Bank, 2013), among the fifteen-year old pupils, the highest result are demonstrated by the schoolchildren from Shanghai, 613 points; Russia, 482 points and the thirty-fourth place; Kazakhstan, 482 points and the forty ninth position out of the sixty-five member countries; the lowest results are by the schoolchildren from Peru, 368 points.

Kazakhstan's results in PISA 2012 in mathematics and science have improved significantly compared with the results of 2009. Since 40 points equal one year of schooling, the increases of achievements in mathematics and science are separately equivalent to more than half a year of schooling.

Kazakhstan has significantly improved the performance in mathematics since the first testing in the year 2009. Despite this improvement, the results of

Kazakhstan in 2012 were below the level of the countries in the region, with which the comparisons are made. Despite notable achievements in mathematics and natural sciences as compared to the first assessment in 2009, Kazakhstan's score is still lower than for similar countries in Europe and Central Asia (ECA), in particular, Turkey and Russia. Kazakhstan lags behind the ECA average in math by 35 points, and the lag from the average scores of OECD is even greater. The gap between the performance of Kazakhstan and the OECD countries is equivalent to one and a half years of schooling in mathematics.

Currently, in Kazakhstan, two international studies evaluating the educational achievements of pupils have been completed: TIMSS-2015 and PISA-2015. The PISA-2015 study involved 7873 15-year-old pupils from 232 educational organizations of 16 regions of the country. In the project TIMSS-2015, Kazakhstan was represented by 9621 pupils from 179 educational institutions, including 4763 fourth-graders and 4858 eighth-graders. The results of international studies TIMSS-2015 and PISA-2015 will be published by OECD and IEA in December 2016.

Research results

In the study, we used the PISA test assignments on mathematical literacy. The pupils were solving the proposed assignments in one of the areas of math, "Changes and relationships" (Algebra: equations, inequalities, equivalence and divisibility).

At the diagnostic stage, five test tasks were proposed to the pupils (Mathematical Literacy of Pupils, 2014).

After completing the test, there were considered the examples of solving exponential inequalities by the method of inverse operations. In accordance with the recommendations of experts, rather than solving three or four different problems, it is useful to solve one problem by several methods. The development of intellectual activity of pupils is possible under the targeted activation of the process of cognition (Sakenov, 2012; Schantz, 2012).

We begin the solution process with two standard exponential inequalities of the form:

$$a^{\varphi(x)} > c \ \ (1), \quad a^{\varphi(x)} < c \ \ (2)$$

We will assume that $\varphi(x)$ is a rational expression. The base of the exponential function can be greater than one or less than one. From the viewpoint of effectiveness of the work, we consider each case separately.

If a > 1, then, solving the inequalities (1), (2) by the inverse operation method, we get:

$$a^{\varphi(x)} > c \Rightarrow \log_a a^{\varphi(x)} > \log_a c \Rightarrow \varphi(x) > \log_a c \quad (3)$$

$$a^{\varphi(x)} < c \Rightarrow \log_a a^{\varphi(x)} < \log_a c \Rightarrow \varphi(x) < \log_a c \quad (4)$$

If we compare the signs of inequalities in the initial and final expressions in each of the relations (3) and (4), we discover that the inequality signs are preserved, since a > 1. Consider now the case a < 1. Let $a = \frac{1}{b}$, whereas $b > 1$. Then $a^{\varphi(x)} > c \Rightarrow \left(\frac{1}{b}\right)^{\varphi(x)} > c \Rightarrow b^{-\varphi(x)} > c \Rightarrow \log_b b^{-\varphi(x)} > \log_b c \; -\varphi(x) > \log_b c \Rightarrow \varphi(x) < -\log_b c.$

Let us connect the initial and final equalities. We have

$$\left(\frac{1}{b}\right)^{\varphi(x)} > c \Rightarrow \varphi(x) < -\log_b c.$$

In the expressions on both sides from the implication sign "$\Rightarrow$", a change of sign in the inequalities took place. The reason for this is that the base of the exponent is less than one. While solving inequalities, one has to be constantly aware of the structure of the bases and change the inequality signs accordingly. And if we solve an inequality with a parameter in the base, it is necessary to consider two cases: *a* < 1 and *a* > 1. In such cases, an information overload of memory occurs.

Henceforth, for the purpose of relieving the memory, we will assume that the base of the exponent is greater than one. If the base in specific inequalities is less than one, then, changing the signs in the exponent, we can always ensure that the base of the power is greater than one.

For example, the inequality $0.(3)^x > 9$ has in its base a number, which is less than one.

$$0.(3)^x > 9 \Rightarrow \left(\frac{3}{9}\right)^x > 9 \Rightarrow \left(\frac{1}{3}\right)^x > 9$$

$$3^{-x} > 3^2 \Rightarrow -x > 2 \Rightarrow x < -2.$$

In this example, the change of sign of the inequalities is not based on a rule, but on the basis of information received by the optic channel.

Consider some examples.

Example 1. $4^x > 5 \Rightarrow x > a$?

Solution. Since 4 > 1, we keep the inequality sign in the requirement. We do not know the lower bound for *x*. Denote it by the letter *a*. Let us solve the inequality by the method of inverse operations.

$$4^x > 5 \Rightarrow \log_4 4^x > \log_4 5 \Rightarrow x > \log_4 5.$$

The numbers 4 and 5 are coprime. Therefore, x is bounded from below by an irrational number, while it is not bounded from above. In mathematics, this idea of "not being bounded from above" is denoted by the sign "∞", called infinity. The word "infinity" has the meaning that the variable quantity x can move upward indefinitely. This movement can be written in the form of inequalities as follows:

$$\log_4 5 < x < +\infty$$

The double inequality can be considered as a set of numbers, enclosed between the numbers $\log_4 5$ and $+\infty$. This set can be written in the form $(\log_4 5,+\infty)$. Thus, the solution of the inequality can be written in two ways:

$$\log_4 5 < x < +\infty \;\Rightarrow\; (\log_4 5,+\infty).$$

In mathematics, the sign "∞" is not openly referred to as a number. However, it is used as such. Let us include it into the set of numbers, calling it improper; however, it will obey different rules of additions and multiplication.

Example 2. $0.(6)^x \le \frac{8}{27} \;\Rightarrow\; x \ge a$?

Solution. Since the base of the power is less than one, the inequality sign in the requirement has been replaced by the reverse one. In order for the reasoning to have logical stability, we move, from the beginning, the base of powers to a number greater than one. Thus,

$$0.(6)^x \le \frac{8}{27} \;\Rightarrow\; \left(\frac{2}{3}\right)^x \le \frac{8}{27} \;\Rightarrow\; \left(\frac{3}{2}\right)^{-x} \le \frac{8}{27},$$

$$\log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{-x} \le \log_{\frac{3}{2}}\frac{8}{27} \;\Rightarrow\; -x \le \log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{3} \le \log_{\frac{3}{2}}\frac{8}{27}$$

$$x \ge -\log_{\frac{3}{2}}\left(\frac{2}{3}\right)^{3} \;\Rightarrow\; x \ge \log_{\frac{3}{2}}\left(\frac{2}{3}\right)^{-3},$$

$$x \ge \log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{3} \;\Rightarrow\; x \ge 3.$$

The answer: $x \in [3,+\infty)$

Example 3. $0.4^{2x} \cdot \left(\frac{125}{8}\right)^{x-1} \geq \frac{2}{5} \Rightarrow x \geq a$?

Solution. In the previous examples, there was one exponential function in the structure of inequalities; and we were guided by one idea during transformations: to reduce the base to a number greater than one. In the structure of this inequality, the amount of information has increased. In such cases, the necessary information is extracted from the structure of the reference. Since each side of the inequality contains one term, then as a guide we take the inequality:

$$a^{\varphi(x)} > c$$

The first information extracted from the reference is such that each factor of this inequality must be represented in the form of power. So,

$$0{,}4^{2x} \cdot \left(\frac{125}{8}\right)^{x-1} \geq \frac{2}{5} \Rightarrow \left(\frac{2}{5}\right)^{2x} \cdot \left(\frac{5}{2}\right)^{3x-3} \geq \frac{2}{5}$$

Comparing the bases of powers, we decide that it is necessary to reduce the base $\frac{2}{5}$ to the base $\frac{5}{2}$.

$$\left(\frac{5}{2}\right)^{-2x} \cdot \left(\frac{5}{2}\right)^{3x-3} \geq \frac{2}{5} \Rightarrow \left(\frac{5}{2}\right)^{-2x+3x-3} \geq \frac{2}{5},$$

$$\left(\frac{5}{2}\right)^{3x-3-2x} \geq \frac{2}{5} \Rightarrow \left(\frac{5}{2}\right)^{x-3} \geq \frac{2}{5} \Rightarrow \log_{\frac{5}{2}}\left(\frac{5}{2}\right)^{x-3} \geq \log_{\frac{5}{2}}\frac{2}{5}$$

$$x-3 \geq \log_{\frac{5}{2}}\frac{2}{5} \Rightarrow x-3 \geq \log_{\frac{5}{2}}\left(\frac{5}{2}\right)^{-1},$$

$$x-3 \geq -1 \Rightarrow x \geq 2.$$

The answer: $x \in [2, +\infty)$

Example 4. $2^{\sqrt{x-1}} - 10 + 16 \cdot 2^{-\sqrt{x-1}} \geq 0 \Rightarrow x \underset{\leq}{\overset{\geq}{}} a$?

Solution. Inequality contains one exponential function. We denote it by y and extract a quadratic inequality from the three-term exponential inequality. We have:

$$y = 2^{\sqrt{x-1}} \Rightarrow y - 10 + 16 \cdot y^{-1} \geq 0,$$

$$y^2 - 10y + 16 \geq 0 \Rightarrow (y-2)(y-8) \geq 0.$$

1) $\begin{cases} y-2 \leq 0 \\ y-8 \leq 0 \end{cases} \Rightarrow y \leq 2.$ 2) $\begin{cases} y-2 \geq 0 \\ y-8 \geq 0 \end{cases} \Rightarrow y \geq 8.$

Replacing y by the exponential function, we obtain the following two standard exponential inequalities. We have:

$2^{\sqrt{x-1}} \leq 2$, (1) $2^{\sqrt{x-1}} \geq 8$. (2)

Since the base of powers is greater than one, these inequalities are solved by the method of inverse operations, while the inequality signs are preserved.

$$2^{\sqrt{x-1}} \leq 2 \Rightarrow \log_2 2^{\sqrt{x-1}} \leq \log_2 2,$$

$$\sqrt{x-1} \leq 1 \Rightarrow x - 1 \leq 1, \Rightarrow x \leq 2.$$

$$2^{\sqrt{x-1}} \geq 8 \Rightarrow \log_2 2^{\sqrt{x-1}} \geq \log_2 8,$$

$$\log_2 2^{\sqrt{x-1}} \geq \log_2 2^3 \Rightarrow \sqrt{x-1} \geq 3,$$

$$x - 1 \geq 9 \Rightarrow x \geq 10.$$

The solution of the first standard exponential inequality is bounded from above, while unbounded from below. Therefore, the solution can be written in the form of the double inequality $-\infty < x \leq 2.$

The solution of the second standard exponential inequality is bounded from below and unbounded from above. The solution has the form $4 \leq x < \infty.$ The answer: $(-\infty;2] \cup [4;\infty)$

Example 5. $2^{\sqrt{x-1}} - 10 + 16 \cdot 2^{-\sqrt{x-1}} \leq 0 \Rightarrow x \overset{\geq}{\underset{\leq}{}} a$?

Solution. We specifically took the same three-term inequality in order to demonstrate the impact of the inequality sign on the structure of the standard exponential inequality. When an exponential function is introduced into the structure of the quadratic equation, then the positive parts of the parabola are mapped into another curve with preservation of each part. In solving the three-term equation, i.e. when denoting the exponential function by a new variable, we distinguish one part of the standard equation and simultaneously distinguish a quadratic equation, from which another part of the standard equation is determined. The same thing happens with the inequality; the boundary curves are only transformed together with the interior. Let us find out, in connection with the sign change in the three-term inequality, which part of the parabola has positive ordinates, and clarify the structure of the standard inequality.

$y = 2^{\sqrt{x-1}}$. Then we get:

$$y^2 - 10y + 16 \le 0 \Rightarrow (y-2)(y-8) \le 0.$$

In this case, we have:

$$\begin{cases} y-2 \ge 0 \\ y-8 \le 0 \end{cases} \Rightarrow \begin{cases} y \ge 2 \\ y \le 8 \end{cases} \Rightarrow 2 \le y \le 8.$$

Thus, the standard inequality has the form: $2 \le 2^{\sqrt{x-1}} \le 8.$

This inequality is solved by the method of inverse operations. We have:

$$2 \le 2^{\sqrt{x-1}} \le 8 \Rightarrow \log_2 2 \le \log_2 2^{\sqrt{x-1}} \le \log_2 8,$$

$$1 \le \sqrt{x-1} \le 3 \Rightarrow 1 \le x-1 \le 9 \Rightarrow 2 \le x \le 10.$$

Example 6. $3^{2x} - 6 \cdot 6^x + 8 \cdot 2^{2x} \le 0 \Rightarrow a? \le x \le b?$

Solution. The quadratic polynomial contains the exponents with different bases. Moreover, the bases of the two extreme powers are prime numbers, whereas the base in the middle term is the product of prime numbers. Based on this information, we decide that it is necessary to divide both sides of inequality by 2^{2x}. Then we get:

$$\left(\frac{3}{2}\right)^{2x} - 6\left(\frac{3}{2}\right)^x + 8 \le 0$$

The structure of the exponential function has been determined.

Using the substitution $y = \left(\frac{3}{2}\right)^x$, we reduce this inequality to a quadratic one:

$$y^2 - 6y + 8 \le 0 \Rightarrow \begin{cases} y-2 \ge 0 \\ y-4 \le 0 \end{cases} \Rightarrow \begin{cases} y \ge 2 \\ y \le 4 \end{cases} \Rightarrow 2 \le y \le 4.$$

Let us write standard exponential inequalities:

$$2 \le \left(\frac{3}{2}\right)^x \le 4.$$

We will solve it by the method of inverse operations. Taking logarithms with respect to the base 3/2, we have:

$$\log_{\frac{3}{2}} 2 \le \log_{\frac{3}{2}} \left(\frac{3}{2}\right)^x \le \log_{\frac{3}{2}} 4,$$

$$\log_{\frac{3}{2}} 2 \le x \le \log_{\frac{3}{2}} 4 \; .$$

The answer: $[\log_{\frac{3}{2}} 2; \log_{\frac{3}{2}} 4]$

Example 7. $98 - 7^{x^2+5x-48} \ge 49^{x^2+5x-49} \Rightarrow b \le ? \ge x \le ? \ge c$

Solution. Let us rewrite this inequality in the following form:

$$7^{2(x^2+5x-49)} + 7^{x^2+5x-48} - 98 \le 0 \Rightarrow 7^{2(x^2+5x-48-1)} + 7^{x^2+5x-48} - 98 \le 0,$$

$$7^{2(x^2+5x-48)} \cdot 7^{-2} + 7^{x^2+5x-48} - 98 \le 0 \qquad \Rightarrow$$

$$7^{2(x^2+5x-48)} + 49 \cdot 7^{x^2+5x-48} - 49 \cdot 98 \le 0$$

$$\left|7^{x^2+5x-48} = y\right| \Rightarrow y^2 + 49y - 49 \cdot 98 \le 0$$

$$\left(y + \frac{49}{2}\right)^2 - 49 \cdot 98 - \left(\frac{49}{2}\right)^2 \le 0 \Rightarrow \left(y + \frac{49}{2}\right)^2 \le \frac{4 \cdot 49 \cdot 98 + 49^2}{4},$$

$$y + \frac{49}{2} \le \pm \frac{49\sqrt{4 \cdot 2 + 1}}{2} \Rightarrow \left(y + \frac{49}{2}\right)_{1,2} \le \pm\left(\frac{49 \cdot 3}{2}\right),$$

$$y_1 + \frac{49}{2} \le -\frac{147}{2} \Rightarrow y_1 \le -\frac{147}{2} - \frac{49}{2} = -98 \Rightarrow y_2 \le 49.$$

The equation $7^{x^2+5x-48} = y$ results in the condition $y > 0$, therefore

$$7^{x^2+5x-48} = 49 \Rightarrow \log_7 7^{x^2+5x-48} = \log_7 49,$$

$$\log_7 7^{x^2+5x-48} = \log_7 7^2 \Rightarrow (x^2 + 5x - 48)\log_7 7 = 2\log_7 7,$$

$$x^2 + 5x - 48 = 2 \Rightarrow x_1 = -10,\ x_2 = 5.$$

The solutions of this equation are -10 and 5. If we substitute it into the original inequality, we obtain 0 = 0. These numbers satisfy the unstrict inequality: $x \in [-10,\ 5]$

After completion of the formative stage of the research, a control sample was extracted with the use of other five test assessments of PISA from the same area (OECD, 2012).

Discussion of the results

For successful solving the test assignments, the pupil needs to possess such competences as formulation, application and interpretation. Formulation is the identification of possibilities for the use of mathematical apparatus. In addition, during the test, it is necessary to find out what important mathematical aspect should be used in the process of analysis. Application is the use of knowledge, concepts and tools of mathematics in solving the tasks. Besides, a pupil must have the ability to simplify the problems so that it becomes possible to use mathematical apparatus for carrying out calculations and algebraic operations. Interpretation is choosing a rational and appropriate mathematical solution for a particular assignment.

Comparison of the results of the diagnostic and control stages of the study showed a significant improvement in the test performance.

The pupils have much better coped with the tasks on the use of mathematical analytical skills.

Conclusion

The developed thinking is of paramount importance for social and private life of every person. The process of thinking is a compass, reasonably guiding the human behavior. It is exactly the intellectual activity training that plays a leading role in the development of intellectual potential and creativity of the young person, in the formation of personality, which must be adapted to the life in the today's ambiguous and contradictory world. Speaking about the personality formation, thinking about an educated, cultured person, one cannot imagine him/her without mathematics education.

The use of the PISA test material on the mathematical literacy gives an idea about the level of development of mathematical thinking and intellectual activity of pupils.

The proposed methods of composing and solving exponential inequalities by the method of inverse operations contribute to the formation of mathematical thinking and development of intellectual skills of pupils. In the independent drawing up the assignments, one uses logical patterns, reveals new links between mathematical facts that promotes creativity in studying mathematics. Also, in drawing up the problems, the ability to control one's own mental activity is developed.

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