LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
2016, VOL. 11, NO. 18, 10761-10775

Homogeneity Assessment of the Leasing Portfolio and Segmentation of Leaseholders for Identifying Risk Concentrations

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ABSTRACT
This paper presents the assessment of homogeneity of the lease portfolio. Based on mathematical transformations, the risk management model of the lease portfolio was developed in accordance with the covariance of default probabilities. A distinctive feature of this model is its practical orientation. Due to the crisis in the global economy, the problem of determining the homogeneity of portfolios and the segmentation of leaseholders has become very relevant. When forming their lease portfolio, leasing companies should take into account the correlation between credit and market risks against the background of the compounding effect. The authors examine the model of the optimum lease portfolio, with a view to creating homogeneous sub-portfolios and taking into account the index of concentration and the correlation of defaults and loans in other segments. It is concluded that in the context of macroeconomic instability, the assessment of portfolio homogeneity and the segmentation of leaseholders helps to establish the most risky sub-portfolio. It will have the highest correlation value, risk concentrations and the average default probability, which, in turn, leads to the greatest standard deviation, and indicates a high level of unexpected losses.

KEYWORDS
Portfolio, leasing, financial risks, default probability, concentration

ARTICLE HISTORY
Received 20 June 2016
Revised 29 July 2016
Accepted 13 October 2016

Introduction

The notion of portfolio is widely used in the management practice of finance companies, especially in the banking sector, primarily for "accumulating" similar loans/investments in the business of leaseholders and managing them as a single large loan. The benefits of such an operation are obvious: management

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costs can be drastically reduced. However, some important points should be taken into account for carrying out such management.

Firstly, it is necessary to monitor the state of the portfolio, and for this, it is necessary to decide which characteristics reflect its quality. By influencing them, one can manage the loan portfolio itself (Suprunovich, 2002; Shapovalov, 2003).

Secondly, it is important to understand that portfolio management is carried out by admission of "suitable" borrowers therein, but it cannot be limited to transactional risk management. For its implementation it is necessary to take into account the characteristics that go beyond a particular loan.

Currently, leasing companies as well as banks do not have and do not use in their system of optimal portfolio formation the method that gives an accurate assessment of the degree of diversification of the loan portfolio and the limits of investment of the leasing company in a business transaction or a borrower. The principle of diversification is the basis for business of any financial institution, which implies a deep understanding of the relationship (correlation) between borrowers, which, in turn, is of crucial importance for many purposes, including such as the establishment of requirements for the leasing company's capital and the pricing of leasing products. At present, portfolio diversification is usually associated with the way institutions conduct their business, for example, by fields of activity, products, etc., taking into account only the accumulated historical experience and experts' data and knowledge. Moreover, there is no technique to calculate in advance the effectiveness of a particular segmentation and, as a result, portfolio diversification in credit institutions (Razumovskiy, 2010).

In the context of the continued macroeconomic instability of the Russian economy, the issues of credit and market risk correlation become most relevant. In this regard, it becomes evident that there is a need to diversify the lease portfolio in order to reduce the impact of the compounding effect on cumulative risks of Russian leasing companies. The solution to this issue should begin with the introduction of modern systems of forming and monitoring the quality of the lease portfolio. The concept of default risk correlation should be at the core of risk models. The models of risk assessment should be based not only on the available data, but also on the economic reality. It must be remembered that there is a positive relationship between loans.

The purpose of this research is to analyze the impact of risk concentrations on the lease portfolio risks, to develop a model for the segmented portfolio and to determine the sequence of actions of the leasing company's risk manager.

Methodological Framework

The compounding effect is the effect of the nonlinear interaction of credit and market risks. For a deeper understanding of the compounding effect, we would like to turn to the conclusions of the Basel Committee on banking. By analyzing the consequences of the global financial crisis of 2008, the working group of the Basel Committee concluded that banks that use the conservative model of risk aggregation, which provides a perfect positive correlation, not always overestimate the risks taken. Moreover, risk assessments with the use of the conservative model may be underestimated by up to 7.5 times at the expense of the so-called compounding effect. The size of the compounding effect is conditioned by the nonlinear interaction of market and credit risks and depends on the degree of market volatility (Sokolov, 2014a,b).

It is evident that the scale of the effect depends on the susceptibility of specific borrowers. Companies with low ratings, being closer to default, are most likely to be at risk of default due to the direct impact of market factors. In other words, the risk of underestimation increases with a reduction in the borrower's rating.

It should be noted that the impact of the compounding effect on the total risk of Russian leasing companies is especially relevant now, when the major systemic risk for all credit institutions is the lack of macrostability.

Therefore, today, in the period of the emergence of new development risks of the Russian economy due to the unstable political and economic situation and the imposition of sanctions on Russia, there is growing attention of leaseholders to the process of portfolio quality monitoring, while in the early 2010s, credit institutions were concentrated mainly on the ground stage - concluding a transaction itself (Demchenko, 2009). However, with regard to the possibility of continuous lease portfolio monitoring itself, these portfolios are incredibly large. What can leasing companies do in such a case? The answer to this question is to reconsider the attitude to the institution of the correlations of market and credit risks, and to start implementing the modern systems of forming and monitoring the portfolio quality.

Let us consider how the credit risk correlation works. As noted above, the principle of diversification is the basis for business of any financial institution and is crucial for establishing the requirements for capital and pricing. Therefore, the correlation concept is at the core of all the risk models, and the assessment of the credit risk correlation (default risk) is the most difficult part of statistical modeling (Kolyasnikova, 2013).

Mistakes in the assessment of correlations may be much more sensitive than mistakes in the assessment of the probability of a leaseholder's default. It was clearly evidenced by the global financial crisis of 2007-2008, which revealed the inability of the majority of risk assessment models to take into account such correlations. When the external impact leads to the correlation of originally independent system components and results in a risk for the whole system, the effect of endogenous correlation factors becomes fully manifested.

In general, as shown in the work of the European Central Bank (European Central Bank, 2007), the default correlation can be both positive – e.g. because companies of the same industry are affected by the same factors (suppliers, raw materials, currency exchange rate) – and negative, when, for example, the elimination of a competitor increases the potential market share of the borrower. At the same time, the correlation determines the extent to which the credits are "migrating" or going into default together.

The credit risk models used are based on the assumption about the conditional independence of defaults, according to which it is understood that the default correlation can be defined by the dependence of all credits in the portfolio on the factors used in the model (Pomazanov, 2012). The position, at which the risk assessment models are based only on the data available, and not on the economic reality, is called "data dependency". This improves the convenience of modeling but at the expense of the loss of accuracy.

X.L. David (2000) in the context of his approach to the correlation noted, “in reality, the default rate for a group of credits tends to be higher in a recession and lower when the economy is booming. This implies that each credit is subject to the same set of macroeconomic environment, and that there exists some form of positive dependence among the credits”.

The Russian school of risk management presented in the library of the World Economic Symposium 2014 by the FEBA-approach, in contrast to most credit risk management models, justified the expediency and the need to use correlations in a wide range: from -1 to +1 (Sokolov, 2014a,b).

Accounting and modeling of the negative correlations used in the factor risk aggregation with regard to the endogeneity of the borrowers’ behavior are required to account for compounding effects and to reduce portfolio volatility (risk) (Aleksandrova, 2014).

The presented approach has been developed taking into account the specifics of the Russian economy and involves the financial institutions’ active use of the information on customers’ foreign trade transactions and cash flows associated with international economic activities. Accordingly, the company’s solvency is sensitive to overall macroeconomic factors, but with a variable correlation sign (Kovalev, 2006).

In addition, there are direct business/legal relations between the companies in the portfolio, which provide a channel of the distribution of financial problems in the portfolio. Such microstructural dependences go beyond the impact of macrofactors on borrowers and may lead to so-called "contagion”. The effect of default contagion may increase the credit risk in the portfolio, i.e. the default of one borrower may cause the default of dependent borrowers. The microstructural interdependence can be both positive and negative (Asyaeva et al., 2016).

D. Egloff, M. Leippold & M. Vanini (2006) completed the studies on large homogenous loan portfolios, which did not address the issues of credit contagion, but analyzed the circular dependence of defaults. In spite of the above-mentioned theoretical principles of the correlation sign, they specified a particular microstructural interdependence solely in the context of the positive correlation. In turn, we find this approach justified because the situation, where the default of one borrower has a direct positive impact on the solvency of another borrower, is quite rare.

As known, the substantiation of correlations within the loan portfolio is part of the concept of risk concentration management. At the same time, with regard to loan portfolio management, it should be noted that currently, there is still no formal technique for measuring the concentration of risk.

Data, Analysis, and Results

In emerging markets, the relationship (correlation) between borrowers is more volatile, which is once again confirmed by the experience of work in the context of sanctions economy. As for the risk of the compounding effect (emergence of the uncontrolled rick concentration), one of the most serious threats to the stability of the financial sector during the crisis is a risk concentration (Sokolov, 2014a,b).

Its significance is confirmed by the Basel Committee, which determines a risk concentration as "any single exposure or group of exposures with the potential to produce losses large enough (relative to a bank's capital, total assets, or overall risk level) to threaten a bank's health or ability to maintain its core operations" (BCBS, 2004). Risk concentrations may represent a significant share of the portfolio credit risk of each financial and credit institution, including leasing companies. Lack of the adequate and proper accounting of risk concentrations may lead to a significant underestimation of the value of the overall credit risk taken by the leasing company, and as a consequence, to an inadequate assessment of the required level of economic capital (Sokolov & Morya, 2012). In view of the recent events, we cannot but agree with the assumption of the Basel Committee that risk concentrations are arguably the most important cause of major problems in credit institutions.

After analyzing the current situation in the banking sector and taking into account the recommendations for banks made by the Basel Committee (BCBS, 2009), we can draw the main conclusions on the portfolio of leasing companies in terms of accounting for financial risks. Leasing companies should have internal policies, systems and control mechanisms in order to identify, measure and monitor credit risk concentrations, to which the company may be subject, both in a narrow and broad sense - concentrations resulting from the interaction of market and credit risks. An example of concentration in a narrow sense is group concentration - risk concentration of a single borrower or a group of related borrowers.

S. Kealhofer & J.R. Bohn (2001) suggested, "Until recently lenders have been reluctant to, or unable to, implement systems for actually measuring the amount of diversification in a debt portfolio". Risk concentrations are determined ex post. The models to quantify concentrations ex ante have not been generally available.

In addition to the fact that the system of the management of credit risk concentrations should be clearly documented, it must include a description of the method for calculating these concentrations and the corresponding limits. The limits should also be defined in relation to the company's capital, and if there are suitable methods of measurement - to the overall level of risk. At the same time, despite all the advantages of setting limits as a percentage of capital, this setting does not provide much information on the actual concentration of loans in the portfolio.

J.M. Diez-Canedo (2002) in his work explored the properties of the "Herfindahl-Hirshman" (HHI) concentration index1. One of the key features of his approach is that "a measure of loan concentration as it relates to risk arises naturally". Russian researcher A. Kadnikov (2012) confirmed this relationship through applied research. With the help of simulation modelling, A. Kadnikov (2012) studied the effect of portfolio concentration on loss distribution (and the credit VaR (Value at Risk). As a result, a variety of approaches to changing the

[1]In accordance with the Methodological recommendations for carrying out the analysis and assessment of competitive environment in the financial services market, the value of HHI index is changed in the range of 0-10,000, with high concentration values corresponding to the index values corridor: 1800<HHI<10,000, moderate concentration values – 800<HHI <1,800; low concentration values – HHI<800.

structure of the portfolio led to the same conclusion. Dependence on HHI index (as a measure of concentration) has the form of a fundamental function (Kadnikov, 2012).

A better understanding of the relationship between the single borrower limit and the concentration index is important for managing risks and regulating the activities of leasing companies.

Traditionally, all credit institutions deal with risk concentration by placing limits on the maximum amount that can be granted to a single borrower, along the various dimensions where concentration can occur, such as by industry, geographic region, loan product, country etc. Normally, "the single borrower limit" is expressed as a proportion "δ" of the capital "K" of the credit institution. However, when discussing group risk concentration, one usually addresses the issue of how much of the total credit exposure is concentrated in a single borrower or a group of borrowers. Turning to the work of J.M. Diez-Canedo (2002), we can see that it focuses on the measurement of concentration in relation to the total value of the loan portfolio, i.e.:

$$f_h \leq \delta K = \frac{\delta K}{V} * V = \delta\psi V = \theta V; h = 1, 2, 3, \dots, N, \tag{1}$$

where θ is the single borrower limit expressed as a proportion of the loan f_h in the total portfolio of loans V;

$\psi = \frac{K}{V}$ is the capitalization ratio.

Therefore, $\theta = \delta\psi$ and the single borrower limit will be expressed as:

$$f_h \leq \theta V; h = 1, 2, 3, \dots, N \tag{2}$$

Then, if it is assumed that all loans are independent and have the same default probability "p", one can define the binary random variable of loss "xi" as:

$$x_i = \begin{cases} f_i \text{ with probability } p \\ 0 \text{ with probability } 1 - p \end{cases} \tag{3}$$

It is clear that $E(x_i) = pf_i$ and Variance $(x_i) = p\,(1 - p)f_i^2$

Since the variables are independent,

$$\mu = E\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N} pf_i = pV, \tag{4}$$

where $V = \sum_{i=1}^{N} f_i$,

$$\sigma^2 = \text{Variance}\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N} \text{Variance}(x_i) = p(1 - p)\sum_{i=1}^{N} f_i^2 \tag{5}$$

Since the distribution of loans f_i is totally arbitrary, it is difficult to know the exact distribution of $\sum_{i=1}^{N} x_i$.

Assume that the required distribution can be approximated by the normal distribution (International Association of Credit Portfolio Managers, 2005), then:

$$\text{VaR}_\alpha = \mu + z_\alpha\sigma = pV + \sqrt{p(1 - p)\sum_{i=1}^{N} f_i^2} \tag{6}$$

If $\text{VaR}_\alpha \leq K$, after making a few mathematical transformations one can arrive at the following expression:

$$\mathrm{H(F)} = \frac{\sum_{i=1}^{N} f_i^2}{(\sum_{i=1}^{N} f_i)^2} \leq \frac{(\psi - p)^2}{z_\alpha^2 p(p-1)} = \Theta(p, \psi, \alpha) \quad (7)$$

Or:

$$\psi \geq p + z_\alpha \sqrt{p(1-p)H(F)} \quad (8)$$

This expression shows how naturally the HHI concentration index emerges, i.e.:

$$\text{Concentration} = \mathrm{H(F)} = \frac{\sum_{i=1}^{N} f_i^2}{(\sum_{i=1}^{N} f_i)^2} \quad (9)$$

Thus, it is seen that VaR directly depends on the concentration level of the portfolio:

$$\mathrm{VaR}_\alpha = \left(p + z_\alpha \sqrt{p(1-p)H(F)}\right)V \quad (10)$$

The following properties for the upper bound $\Theta(p, \psi, \alpha)$ of the concentration index H(F) play an important role in the application of the above calculations for risk management (Diez-Canedo, 2002):

1. $\Theta(p, \psi, \alpha)$ varies in direct proportion to the capitalization ratio ψ and inversely to the default probability "p" and the value VaR at risk confidence level.

2. If the concentration degree exceeds the upper bound (i.e. $H(F) > \mathit{\Theta}(p, \psi, \alpha)$, then the capital of the leasing company is at risk, for the given confidence level.

3. If the default probability "p" exceeds the capitalization ratio, then the capital of the bank is at risk for any confidence level, regardless of the concentration of the loan portfolio.

4. If $\Theta(p, \psi, \alpha) > 1$, then no degree of concentration places the capital of the bank at risk.

Therefore, the first point is seen from the expression of $\Theta(p, \psi, \alpha)$. The second point can be easily verified if $H(F) > \mathit{\Theta}(p, \psi, \alpha)$, then,

$$\mathrm{VaR}_\alpha = \left(p + z_\alpha \sqrt{pqH(F)}\right)V > \left(p + z_\alpha \sqrt{pq\Theta}\right)V = \left(p + \frac{z_\alpha(\psi - p)\sqrt{pq}}{z_\alpha\sqrt{pq}}\right)V = K, \quad (11)$$

where $q = 1 - p$.

The third point can also be easily verified.

$$\mathrm{VaR}_\alpha = \left(p + z_\alpha \sqrt{pqH(F)}\right)V > \left(\psi + z_\alpha \sqrt{pqH(F)}\right)V = K + z_\alpha \sqrt{pqH(F)} > K \quad (12)$$

As for the fourth point, it is well known that

$$\mathrm{H(F)} = \frac{\sum_{i=1}^{N} f_i^2}{(\sum_{i=1}^{N} f_i)^2} < 1 \text{ for any F.}$$

The above properties provide some useful rules for the risk manager and for the regulator. First, they help to determine the adequacy of capital "because one obtains precise measures of the adjustments in the capitalization ratio required by variations in the default rates and/or the concentration of the loan portfolio. Furthermore, depending on the amount of control that banks have on the default ratio and loan concentration, adjustments in the default probability and the concentration of the loan portfolio necessary to maintain capital adequacy can also be calculated. Thus, if the concentration of the loan portfolio exceeds the bound at the desired confidence level, inequality (…) provides a convenient means of fine tuning the adjustments required in *ψ*, *p* and *H(F)* so that credit risk does not place the capital of the bank in jeopardy. Also interesting, is that if the default rate of the portfolio exceeds the capitalization ratio, a signal of alarm is sent to the risk manager and the financial authorities, that the banks' capital is at risk regardless of the concentration of the loan portfolio and the confidence level adopted" (Diez-Canedo, 2002).

In order to understand the cause of dependence, according to which the greater amount of loans in fewer borrowers, the greater concentration, it should be understood that maximum concentration occurs when all loans belong to a single borrower, and minimum concentration occurs when all borrowers have the same amount.

To further explore the properties of the index, J.M. Diez-Canedo (2002) used F as the vector of loans $f_h \geq 0$ for $h = 1, 2, 3, \ldots, N$. He determined and proved the following properties of the concentration index:

1. If one of the loans is increased by the amount equal to the reduction of the smaller loan, the concentration index of the resulting portfolio will be higher than the concentration index of the original portfolio. Similarly, if the bigger loan is reduced by the amount of increase of the smaller loan, the portfolio concentration index will decrease.

2. If the entire portfolio is concentrated in the minimum number of borrowers, taking into account the limit $f_h \leq \theta V$, then $H(F) \leq 0$.

3. When the limit is placed on each loan as a percentage of the portfolio value, the same limit θ is placed on the concentration measured by the HHI index. Therefore, it is easy to verify the adequacy of capital:

$$\theta \leq \frac{(\psi - p)^2}{z_\alpha^2 p(p-1)} = \Theta(p, \psi, \alpha) \tag{13}$$

The above inequality gives a relatively simple way to verify capital adequacy without making complicated calculations.

Thus, it can be noted that the results obtained can be very useful for the management of group risk concentrations, as far as they provide clear formulas to measure risk, which makes it possible to carry out a detailed quantitative analysis of the necessary regulatory measures for maintaining capital adequacy.

As part of the periodic assessment of the company's systematic risk concentration, HHI may be used in the context of the industry - the industry index, the concentration index by type of provision, the factor concentration index. The value of the industry index, for example, is defined as the sum of squares of the outstanding loan shares of all sectors (for industry concentration) in the total amount of the corresponding aggregate indicators of the leasing company as a whole or in a particular region. The index takes the value from 0 (when an infinite number of branches are covered, each of which accounts for a very small share of the loan portfolio) to 1 (when all loans are concentrated in the same industry).

To compare the various dimensions by concentration, it is advisable to perform the normalization of each index so that the index of dimensions varied in the range of 0-1. Bringing the calculated indices in the normalized view will be carried out using the following expression:

$$HHI_{norm} = \frac{n * HHI - 1}{n - 1}, \tag{14}$$

where n — number of groups in the dimension.

Such a possibility of the concentration level operative control, combined with a relatively simple way to verify capital adequacy without making complicated calculations, helps to carry out the quantitative analysis of the necessary measures for regulating capital adequacy.

In this model of risk management, certain assumptions were made (Adelman, 1996):

– the portfolio loss distribution can be characterized by its mean and its variance;

– the loans included in the measurement, by which concentration is possible, have equal and independent default probabilities;

– only one measurement of loan concentration in the portfolio is possible;

– the level of losses in the event of default is 1.

Further study of risk management of the lease portfolio is connected with the removal of these restrictions.

Thus, dangerous risk concentrations with a clear behavioral content can be identified ex ante in the portfolios, formed in accordance with the causal factor. Currently, the causal models, unfortunately, are used very rarely in credit risk management (Ajupov, Mishina & Ivanov, 2014).

As shown at the World Economic Forum in Davos (2013), the dynamics of prices for hydrocarbons is the most important exogenous risk factor for the scenarios of Russia's economy. In the scenarios prepared by the expert group of the Russian economy, illustrated by the state of matryoshka dolls, this is the main risk factor. At the same time, all the scenarios are inherently negative, including the Precarious Stability scenario (a sudden sharp and sustained drop in energy prices) (matryoshka is breaking down) and the Beyond Complacency scenario. In the latter case, high-energy prices persist, and matryoshka doll, symbolizing basis, is happy and littered with icons of major currencies, but another matryoshka is hanging over darkly, apparently, more well-informed about the real situation. It should be noted that high oil and gas prices account for the strengthening of the ruble, which, in turn, is a major prerequisite for the development of so-called "Dutch disease" of the economy.

Unlike the micro-level models, which are based on the forecast of prices for raw materials and have a low discriminatory ability, such a causal factor as an exchange rate provides the possibility of separating highly desirable default negative correlations in portfolio management. Perhaps this is why the 2012 Global Economic Symposium (Rio de Janeiro) included the paper of the representative of the Russian school of risk management (Sokolov, 2009) in a number of promising developments in the framework of The Future of Global Financial Governance.

Indeed, financial risks are determined not only by the level of risk components but also by the degree of their interrelation. If synchronization for many processes is the aim and achievement, the synchronization of defaults is the main threat to any financial institution.

Given the assumptions in the risk management model, which have been adopted earlier, consider a generalized model with a phased easing of the assumptions mentioned above, which was described by J.M. Diez-Canedo (2002).

Assume that the portfolio loss distribution can be determined by its mean and its variance and that the vector of default probabilities π and the co-variance matrix M are calculated exogenously. Developing the same line of the previous calculation, we receive:

$$VaR_\alpha = \pi F + z_\alpha \sqrt{F^T M F} \leq K \quad (15)$$

As far as M is a positive definite matrix, there exists a matrix Q:

$$M = Q \Lambda Q^T \quad (16)$$

where Λ is a diagonal matrix of the characteristic values of M;

Q is an orthogonal matrix of the eigenvectors of M provided that $Q^{-1} = Q^T$.

Let $S = Q\sqrt{\Lambda}Q^T$, where $\sqrt{\Lambda}$ is a diagonal matrix of the square roots of the characteristic values of M, then $M = S^T S$.

Now change the variable to $G = SF$ so that $F^T M F = G^T G$. This change of variables rescales the vector F, i.e. it rescales the loans in the portfolio given to leaseholders according to the covariances of default probabilities between the loans, so that the loans with higher loss covariances will increase in size, while the loans with smaller loss covariances will decrease.

Thus, despite the fact that much credit in one hands is potentially dangerous, it is even more dangerous when too much risk is concentrated in a particular group of borrowers. As a result, a highly diversified portfolio of small loans that are highly correlated and have high variances may be riskier than a small portfolio of large loans that are uncorrelated and have low default probabilities.

Further, generalizing the expression for the capitalization ratio, we get:

$$\psi = \frac{K}{V} \geq \frac{VaR}{V} = \bar{p} + z_\alpha \sqrt{\frac{F^T M F}{F^T F} H(F)} = \bar{p} + z_\alpha \sigma \sqrt{H(F)}, \quad (17)$$

where $\sigma^2 = \frac{F^T M F}{F^T F} = R(F, M)$ is a Rayleigh's Quotient, a measure of the standard deviation of losses;

$\bar{p} = \frac{\pi^T F}{V}$ is the expected loss of the portfolio relative to its value;

$V = 1^T F$ is the portfolio value.

The limit on concentration and the single borrower limit will be expressed as:

$$H(F) \leq \theta \leq \left(\frac{\psi - \bar{p}}{z_\alpha \sigma}\right)^2 \quad (18)$$

It should be noted that the total variance of losses $\sigma\sqrt{H(F)}$ is decomposed into the variation-covariation effect, expressed as σ, and the concentration level H(F). Therefore, the resizing of the loan vector through the co-variance matrix M implies that concentration in the number of loans is not necessarily a good measure of risk concentration.

Examine concentration more detailed in the case of the non-segmented portfolio. In order to illustrate how correlation affects the level of concentration and increases risk, consider the example when all loans have the same default probability p, and each pair of loans is similarly correlated through a correlation coefficient ρ. In this case, the covariance between any two loans (i, j) is (Bernoulli distribution):

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} = \sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)}\rho_{ij} = p(1-p)\rho \quad (19)$$

In this case, the covariance matrix has the following structure:

$$M = p(1-p)\begin{pmatrix} 1 & p \cdots & p \\ \vdots & \ddots & \vdots \\ p & \cdots p & 1 \end{pmatrix} \quad (20)$$

It is convenient to represent M in the following form:

$$M = p(1-p)\{\rho 1 1^T + (1-\rho)I\}, \quad (21)$$

where "1" is a unit column;

I is a single matrix.

Therefore, the variance of portfolio losses is:

$$F^T M F = p(1-p)\{\rho(1^T F)^2 + (1-\rho)F^T F\}. \quad (22)$$

Then,

$$VaR_\alpha = V\left\{p + z_a\sqrt{p(1-p)}\,\sqrt{\rho + (1-\rho)H(F)}\right\} \quad (23)$$

As seen in the expression received, full variance is decomposed into two parts: the first is the Bernoulli variance and the second is related to concentration:

$$H' = \rho + (1-\rho)H(F) \quad (24)$$

It should be noted that when correlation is positive, H' can be interpreted as a combination between a totally concentrated portfolio (H' =1) and a portfolio with the concentration level H(F). It is evident that H' increases with ρ, and for $\rho = 0$ we have H'=H(F), while for $\rho = 1 - H'$=1. In other words, if all the portfolio loans are absolutely and positively correlated, then in terms of risk they behave as a large single loan. In general, it can be said that the portfolio with correlation between loans behaves exactly the same as the portfolio without correlation, but with the concentration index H' instead of H(F). Therefore, H' could be understood as a correlation-adjusted concentration index, or more briefly - risk concentration.

Moreover, the result received can be used to calculate the index H' for any portfolio by computing p and ρ in advance:

$$p(1-p)[\rho + (1-\rho)H(F)] = R(M,F)H(F) \quad (25)$$

Then

$$\rho = \frac{[R(M,F) - p(1-p)]H(F)}{p(1-p)[1-H(F)]} \quad (26)$$

However, credit and financial institutions usually divide their loan portfolios into sub-portfolios in accordance with a particular criterion, which in some way is connected with the way they organize their business. It is particularly desirable to apply other criteria for the purposes of credit risk in general and concentration in particular. One of the most difficult problems is to determine the potentially dangerous concentration segments ex ante, and it may have nothing to do with the organizational structure of the leasing company (Sokolov, 2012). The described model helps to identify these potentially dangerous segments.

Assume that the portfolio F is split into h segments, $F^T = (F_1, \dots, F_h)$, where F_i is a vector whose elements are the amounts of loans in the segment i. Consider also the vector of default probability and the covariance matrix:

$\pi = (\pi_i)$, where π_i is the vector of default probability for the segment i,

I = 1, 2, 3, …, h;

the covariance matrix is expressed as follows:

$$M = \begin{bmatrix} M_1 & C_{12} & \dots C_{1h} \\ C_{21} & M_2 & \dots C_{2h} \\ \vdots & \vdots & \ddots \; \vdots \\ C_{h1} & C_{h2} & \dots M_h \end{bmatrix}$$

(27)

Each diagonal block M_i is the covariance matrix for the loans in the segment i and has dimension $(N_i \times N_i)$, where N_i is the number of loans in the segment. The matrices C_{ij} are the covariances of the defaults between the loans of the segments i and j.

Let $V_i = \sum_{j \in F_i} f_j$ be the value of the portfolio of the segment i and $\sum_{i=1}^{h} V_i = V$. Let $K_i = \gamma_i K$, where $\gamma_i = \frac{V_i}{V}$ is the proportion of capital, allocated to the segment $i, \gamma_i \in [0,1] \forall i, \sum_{i=1}^{h} \gamma_i = 1$.

Note that the analysis of individual segments should take into account only correlations between defaults of the loans in the segment i with the loans of the other segments, while other correlations are irrelevant. Thus, from the matrix M build the matrix S_i having the following structure:

$$S_i = \frac{1}{2} \begin{bmatrix} 0 & \dots & C_{1i} & \dots & 0 \\ \vdots & & \vdots & \dots & \vdots \\ C_{i1} & & 2M_i & \dots & C_{ih} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & C_{hi} & \dots & 0 \end{bmatrix}$$

(28)

Note that $\sum_i S_i = M$.

It is important that when combining the results of the analysis of individual segments and the results of the overall portfolio, the relative weight of each segment in the overall portfolio does not distort the results of the portfolio as a whole. The property of additivity is necessary to ensure that addition by individual segments is consistent with the overall portfolio.

It means that capital adequacy for the overall portfolio would be as follows:

$$\psi = \sum_{i=1}^{h} \gamma_i \psi_i,$$

(29)

where ψ_i is capital adequacy for an individual segment.

Introduce the following coefficient:

$$\phi = \frac{\sqrt{F^T MF}}{\sum_{i=1}^{h}\sqrt{F^T S_i F}} \quad (30)$$

With regard to previous statements, VaR$_i$ for an individual segment will be:

$$v_i = \pi_i^T F_i + z_\alpha \phi \sqrt{F^T S_i F} \le K_i = \gamma_i K \text{ для } i = 1, 2, \dots, h \quad (31)$$

Thus, $\sum_i v_i = VaR_\alpha = \pi^T F + z_\alpha \sqrt{F^T MF}$ (32)

Dividing by V_i will result in capital adequacy for an individual segment:

$$\psi_i \ge \frac{v_i}{V_i} = \overline{p_\iota} + z_\alpha \phi \sqrt{R_i(F_i, M_i) H(F_i) + \frac{1}{(1^T F_i)^2} \sum_{\{j/j \ne i\}} F_i^T C_{ij} F_j} \quad (33)$$

Solving this inequation with respect to $H(F_i)$, receive:

$$H(F_i) \le \left(\frac{\psi_i - \overline{p_\iota}}{z_\alpha \phi \sigma_i}\right)^2 - \frac{1}{(\sigma_i V_j)^2} \sum_{\{j/j \ne i\}} F_i^T C_{ij} F_j \quad (34)$$

where $\sigma_i = \sqrt{\frac{F_i^T M_i F_i}{F_i^T F_i}} = \sqrt{R_i(F_i, M_i)}$, and the single borrower limit is $\theta_i \le H(F_i)$.

It is interesting to note that the limit on the concentration index thus contains a correction for default correlation with the loans in other segments.

Conclusion

The use of the model for the segmented portfolio comes down to the following algorithm of actions of the leasing company's risk manager:

1. To determine the vector of default probabilities for all sub-portfolios and the covariance matrix, in which the idiosyncratic covariance matrices are arranged diagonally for all sub-portfolios, respectively, and other elements – the covariance matrices between the loans of sub-portfolios i and j.

2. To determine the value of sub-portfolios, the weights of each individual segment, capital allocated to this segment.

3. To calculate the corresponding concentration indices (H(F)), the risk concentration indices $\boldsymbol{H'}$, the ratio of risk concentration to the level of concentration HHI, the default probability p, the correlation coefficients ρ and the variance $\boldsymbol{\sigma}$ for each individual segment. Having this information, one can verify all the conditions of capital adequacy.

These data illustrate the relationship between the default probability and the variance for each individual segment. By analyzing them, one can see the most risky sub-portfolio – it will have the greatest correlation value, risk concentrations and the average default probability, which leads to the highest standard deviation, which in turn would indicate a high level of unexpected losses.

If there are limitations in the use of a common model without the possibility to carry out an analysis of individual segments, any sub-portfolio could go unnoticed. It is also clear that the result depends on the method of segmentation, since it is possible to divide the portfolio in such a way that all the segments will meet the criteria of adequacy, and risk groups will also remain undetected. However, one can determine the state of the portfolio and the degree of concentration ex ante, by trial and error in the worst case.

In many ways, this conclusion is consistent with the principles set out in "Sound Practices in Credit Portfolio Management", which also focused on the fact that financial institutions, in addition to defining the limits of risk concentration, should take into account the effect of correlations between the various factors influencing changes in the level of concentration in the credit portfolio.

Disclosure statement

No potential conflict of interest was reported by the authors.

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