LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
2016, VOL.11 , NO.18,11593-11610

Interference of Spherical Laser Radiation in a Crystalline Compound Lens

Seitbek B. Kyanishbayev[a], Abilhan U. Umbetov[a], Aisaule E. Duisebekova[a], Makhabbat Zh. Umbetova[a], Kamar S. Schongalova[a], Zhanar E. Akhatova[a], Perizat S. Azhibekova[a] and Aigerim Sh. Sokabayeva[a]

[a]Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, KAZAKHSTAN

ABSTRACT

The operation of a crystalline lens is based on the phenomenon of birefringence in crystals belonging to such compound lenses in various combinations. Crystalline systems are typically used in the form of two types of prisms: prisms giving one linearly polarized beam at the output (polarizing prisms), and prisms giving two beams that are polarized in two mutually perpendicular planes (birefringent prisms). This article discusses the types and properties of polarized waves (laser radiation) arising from the propagation of convergent laser radiation through a crystalline compound lens (CCL). The CCL is formed by a variety of birefringent prisms (prisms of Wollaston, Rochon, Sénarmont etc.), consisting of two wedges of uniaxial crystals. At normal incidence of laser radiation on the input face of the CCL there is a shift between the wavefronts of ordinary (o) and extraordinary (e) beams at the output of the CCL. The superposition of these waves at the output of the CCL results in the emergence of an interference pattern that can be used in a variety of laser polarized interferometers. When the collimated laser beam is incident on the input face of the CCL at an arbitrary angle, four waves are formed at its output. This article discusses the condition for the emergence of these four polarized waves at the output of the CCL, presents the expressions describing each wave, studies the condition for the emergence of interference patterns, types of these interference patterns, and compares the results of theoretical calculations with the experimental data.

KEYWORDS
Crystal, lens, birefringence, polarized beam, interference, laser radiation, ordinary, extraordinary, uniaxial, path difference, analyzer, intensity vector, wave vector, collimated beam

ARTICLE HISTORY
Received:20 August 2016
Revised:28 October 2016
Accepted:15 November 2016

Introduction

There are a large number of varieties of birefringent prisms, whose properties are determined by a relative orientation of the optical axes of crystals

CORRESPONDENCE Abilhan U. Umbetov ✉ Umbetov.a@mail.ru

constituting the prism. The changes in their relative orientation make it possible to obtain systems with various properties. The design of the CCL is based on the operation of the Wollaston prism, the inclined wedge planes of which are replaced by a spherical surface making it possible to shape the convergent polarized beams at the output of the CCL, thereby creating new and exciting opportunities in the use of crystal systems. The known (Born & Wolf, 1999; Landsberg, 1976) calculation methods of the beam path in birefringent prisms are limited to the case of normal incidence of light on the input face of the prism and held separately for each specific prism. It is more complicated to calculate the propagation of waves through the crystal system in the form of a compound lens, which presents a uniaxial crystal.

This paper studies the method of calculating the propagation of spherical laser radiation through the CCL, the conditions for the emergence and transformation of different waves (o-ordinary and e-extraordinary) on the faces of the CCL (input, output and spherical), and presents the expressions describing the properties of polarized waves at the output of the CCL. The authors explore the mode of laser radiation interference at the output of the CCL and obtain the expressions that reflect all the characteristics of interference patterns. The paper shows the feasibility of using polarized beams with the help of the CCL: the interference of polarized beams without the help of the analyzer.

In this regard, the purpose and contribution of this research is as follows: to analyze the CCL in the mode of spherical laser radiation, to obtain the expressions describing the properties of the waves generated at the boundaries and at the output of the CCL as well as to study the properties of polarized waves at the output of the CCL in the interference mode.

The following tasks have been set in the paper:

- to study the propagation of spherical laser radiation (convergent and divergent) through crystalline compound lenses;

- to define the varieties of generated and converted electromagnetic waves on the input, spherical and output lens faces;

- to obtain the expression describing the path of laser radiation in the inside and at the output of the CCL;

- to study the interference of polarized waves at the output of the CCL with and without the use of the analyzer;

- to obtain the expressions describing the interference of polarized laser radiation at the output of the CCL;

- to compare the results of theoretical calculations provided with the experimental data.

The calculation of the propagation of electromagnetic waves in anisotropic crystals, the laws of reflection and refraction on the boundaries of the environmental section were studied by F.I. Fedorov and V.V. Filippov (1968; 1971a; 1971b; 1976), F.I. Fedorov and A.F. Konstantinova (1962) and Fedorov and T.L. Kotyash (1962). However, a rigorous calculation of laser radiation in the system, consisting of a few anisotropic crystals, results in cumbersome expressions, which are little suitable for engineering calculations and do not allow the general properties of the crystalline lens to be investigated. There is a method for calculating the beam path in birefringent prisms of the variable angle (Javan, Bennett Jr. & Herriot, 1961). It seems, however, more complicated to calculate the propagation of waves through the crystal system in the form of a compound lens, which presents a combination of two lenses made of a single crystal and bonded by their spherical surfaces. In the literature, there are practically no detailed studies of crystalline compound lenses, although some issues of the application of such crystalline systems in optical instrument engineering and in a number of laser devices were addressed in works of F.T. Yu (1973), R.J. Collier, C.B. Burkhardt and L.H. Lin (1971), L.M. Soroko (1971), G.V. Dreytsen, Yu.P. Ostrovoskin and E.P. Shedeva (1972).

In laser devices, crystalline systems are used in two basic modes: the mode of the spatial separation of the narrow light beam with orthogonal polarizations and the mode of the interference of light beams at the output of the system.

The first mode is used to control laser radiation, to select the modes of optical resonators (Kulcke et al., 1965; Smith, 1972).

The second mode of crystalline systems underlies the operation of polarization shearing interferometers (Galperin, 1959). The light beam is divided by the system into two orthogonally polarized beams, the fronts of which are shifted relative to each other.

When the beams are shifted by using the analyzer, an interference pattern in the place of their transposition emerges. The form of the pattern and its contrast helps analyze the characteristics of the incident beam, the wavefront curvature and the degree of radiation coherence (Ostrovskiy, 1968). Many accurate measurements, the role of which is rapidly increasing in modern science and technology, are carried out by the interference method. The application area of interference devices has grown significantly due to the creation of new light sources (lasers) and the development of electronics.

Results and Discussion

The purpose of this research is to obtain the expressions describing the propagation of spherical laser radiation in a crystalline compound lens and the interference of polarized beams at the output of the CCL.

The following tasks have been set: to analyze the process of the transformation of electromagnetic waves on the input, spherical and output faces of the CCL as well as to study the behavior of polarized beams at the output of the CCL and the condition for the emergence of an interference pattern.

When the collimated beam is incident at an arbitrary angle, four waves are formed at the output of the CCL. An interesting feature may be noted here: an interference pattern at the output of the CCL is formed without the use of the analyzer. This feature is associated with the effect of the pair combination of four waves at the output of the CCL, while the polarization state of waves within each pair is the same, and both pairs of waves are orthogonally polarized. Each pair of waves when they overlap gives a spatially non-localized interference pattern that can be described using the method outlined above. This feature in the interference of polarized beams appear in the CCL (Figure 1) in the case when the plane z=0 is located on the concave spherical section surface, and the vectors defining the direction of the optical axes of the crystals $\vec{a}_1$ and $\vec{a}_2$ have the following form:

$$\vec{a}_1 = (0, 0, 1),\ \vec{a}_2 = (0, 1, 0).$$

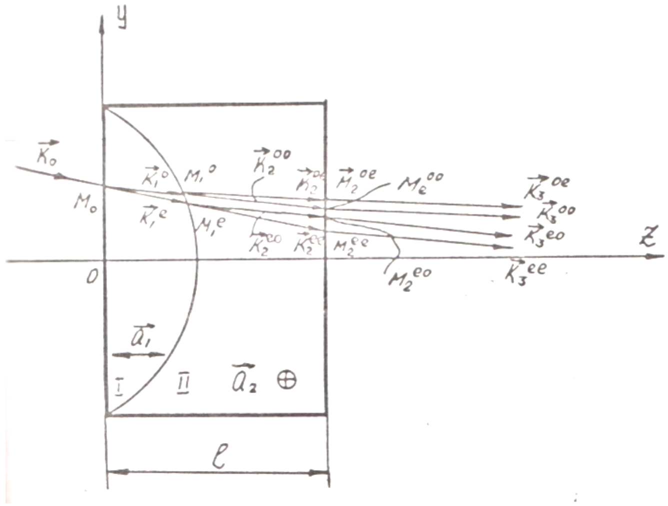


Figure 1. Scheme of the propagation of polarized rays through the CCL in the collimated laser beam incident on the input face at an arbitrary angle ("without-analyzer" interference mode)

The waves will be denoted by the indices (oo), (oe), (eo) and (ee), which mean changing the state of polarization of the waves as they move across the spherical boundary of the CCL section. Consider each of the four waves.

a) Wave (oo):

Suppose that the collimated beam of light is incident on the CCL at an arbitrary angle. The field in this beam can be represented in the form of plane waves with a unit wave vector (Figure 1):

$$\vec{\kappa}_0 = (\sin\alpha\,; 0;\ \cos\alpha); \qquad (1)$$

Select a sufficiently narrow ray of the collimated beam, which intersects z=0 at the point $M_0(d\cos\varphi;\ d\sin\varphi; 0)$ (Figure 2). The (oo) wave is "insensitive" to the inner spherical surface and passes through the CCL as through an isotropic parallel plate with the refractive index n_0. A simple calculation gives the following coordinates of the point M_1^0 at the output of the partial beam:

$$x_1^0 = l tg\alpha_1 + d\cos\varphi;\ y_1^0 = d\sin\varphi; \quad z=l; \qquad (2)$$

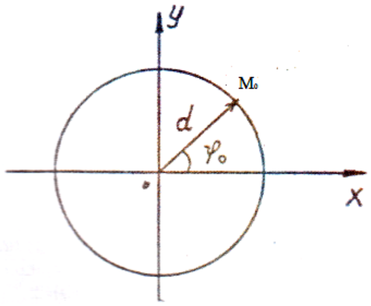


Figure 2. Location of the point M and parameters in the XY plane

b) Wave (eo):

Suppose that the partial beam with a wave vector (1) intersects z=0 at the point $M_0^e(d_e \cos\varphi_e\,;\ d_e \sin\varphi_e\,;0)$, which is generally not coincident with M_0^0. The law of refraction at z=0 has the following form:

$$sin^2\alpha\left(\frac{sin^2\alpha_1^e}{n_e^2}+\frac{cos^2\alpha_1^e}{n_0^2}\right)=sin^2\alpha_1^e; \tag{3}$$

where α_1^e is the angle between the unit wave vector of the refracted wave and the axis z, whence:

$$\mathrm{tg}\alpha_1^e=\frac{n_e \sin\alpha}{n_0\sqrt{n_e^2-sin^2\alpha}}sin^2\alpha; \tag{4}$$

The beam vector determining the partial beam path lies in the plane of the vectors $\vec{K}_0$ и $\vec{K}_1^e$ and forms an angle θ with the axis z:

$$\mathrm{tg}\theta=\frac{n_0^2}{n_e^2}\mathrm{tg}\alpha_1^e; \tag{5}$$

or

$$\mathrm{tg}\theta=\frac{n_e \sin\alpha}{n_0\sqrt{n_e^2-sin^2\alpha}}; \tag{6}$$

Hence, the equation of the (eo) beam trajectory in the field 1 of the CCL has the following form:

$$\frac{x-d_e\sin\varphi_e}{\sin\theta}=\frac{y-d_e\sin\varphi_e}{0}=\frac{z}{\sin\theta}; \tag{7}$$

which makes it possible to find the coordinates of the point M_1^e (Figure 1):

$$x_1^e=\frac{n_0\sin\alpha}{n_e\sqrt{n_e^2-sin^2\alpha}}\frac{l}{2}+d_e\cos\varphi_e;\ y_1^e=d_e\cos\varphi_e;\ z_1^e=\frac{l}{2}; \tag{8}$$

In turn, the refractive condition of the (eo) beam on the spherical surface at the point M_1^e is as follows:

$$[1-(\vec{\kappa}_1\vec{n}_R)^2] = n_0^2\left(\frac{sin^2\alpha_1^e}{n_e^2}\frac{cos^2\alpha_1^e}{n_0^2}\right)[1-(\vec{\kappa}_2\vec{n}_R)^2]; \qquad (9)$$

where $\vec{n}_R = \left\{\frac{d_e\cos\varphi_e + l\,tg\theta}{R};\ \frac{d_e\sin\varphi_e}{R};1\right\}$ is a normal to the spherical surface at the point M_1^e

The wave vector $\vec{\kappa}_2$ is defined in the following form:

$$\vec{\kappa}_2 = \eta\vec{n}_R + \mu\vec{\kappa}_1; \qquad (10)$$

where η and μ are coefficients.

The expression (10) is the condition of coplanarity of the wave vectors of the incident and refracted wave and the normal to the spherical surface at the point M_1^e.

By substituting (10) into (9) and taking into account (6), find:

$$\mu = \frac{1}{n_0 n_e}\sqrt{n_0^2 n_e^2 - (n_0^2 - n_e^2)sin^2\alpha}; \qquad (11)$$

$$\eta = \eta_0\left(1 - \frac{\mu q}{\sqrt{1-\frac{sin^2\alpha}{n_0^2}}}\right) = \eta_0(1-q'),$$

where

$$q = sin^2\alpha_1^e\frac{d_e\cos\varphi_e + l\,tg\theta}{R};$$

$$q' = \frac{\frac{\sin\alpha}{n_0}\,\frac{d_e\cos\varphi_e + l\,tg\theta}{R\sqrt{1-\frac{sin^2\alpha}{n_0^2}}}}{};$$

$$\eta_0 = \sqrt{1-\frac{sin^2\alpha}{n_0^2}} - \sqrt{1-\frac{sin^2\alpha}{n_e^2}}; \qquad (12)$$

Thus, the components of the wave vector $\vec{\kappa}_2^{eo}$ in the paraxial approximation $\left(\frac{d}{R} \ll 1; \frac{l}{R} \ll 1\right)$ can be written as:

$$\kappa_{2x}^{eo} = \frac{\sin\alpha}{n_0} + \eta_0\frac{d_e\cos\varphi_e + l\,tg\theta}{R};$$

$$\kappa_{2y}^{eo} = \eta_0\frac{d_e\sin\varphi_e}{R}; \qquad (13)$$

$$\kappa_{2z}^{eo} = \sqrt{1 - \frac{sin^2\alpha}{n_0^2}} - \eta_0 q';$$

The expressions (13) help to find the equation of the (eo) beam in the field 11 of the CCL

$$\frac{d_e \cos\varphi_e + l tg\theta - x}{\vec{\kappa}_{2x}^{eo}} = \frac{d_e \sin\varphi_e - y}{\vec{\kappa}_{2y}^{eo}} = \frac{l - z}{\vec{\kappa}_{2z}^{eo}}; \quad (14)$$

and to determine the coordinates of the point M_2^{eo}

$$x_2^e = \frac{l}{2}\left(\frac{n_0 \sin\alpha}{n_e\sqrt{n_e^2 - sin^2\alpha}} + \frac{\sin\alpha}{\sqrt{n_0^2 - sin^2\alpha}}\right) + d_e \cos\varphi_e;$$

$$y_2^e = d_e \cos\varphi_e; \quad (15)$$

$$z_2^e = l;$$

The trajectories of (oo) and (ee) beams found make it possible to find the optical path difference between these beams:

$$\Delta Z = \left[\sqrt{\left(1 + \frac{n_0^2 \sin\alpha}{n_e(n_e^2 - sin^2\alpha)}\right)\left(n_0^2 - \frac{n_0^2 - n_e^2}{n_e^2} sin^2\alpha\right)} - \frac{n_0^2}{\sqrt{n_0^2 - sin^2\alpha}}\right]; \quad (16)$$

The condition of interference maxima at the output of the CCL is written as follows:

$$\Delta Z = (\vec{\kappa}_2^{eo} - \vec{\kappa}_2^{oo})\vec{r} = S\lambda, \quad (17)$$

where S=0;±1;±2; r(x, y) is the radius of the point of observation z=l;

The expression (17) includes the difference between the wave vectors of the beams of both polarizations, the components of which include the azimuth angles setting (oo) and (eo) beams at the input of the CCL. These components can be excluded from (17) using the equations (7) and (15), in which it is necessary to let $X_1^e = X_2^e = X$, $y_1^e = y_2^e = y$. The exclusion of the component leads to the following equation of the locus of the interference pattern maxima points:

$$\frac{\eta_0}{R}(X^2 + y^2) - \eta_0 \frac{l}{2R} \frac{\sin\alpha}{\sqrt{n_0^2 - sin^2\alpha}} = s\lambda - \Delta \quad (18)$$

or

$$(X - B)^2 + y^2 = R^{*2}, \quad (19)$$

where $B = \frac{l}{4} \frac{\sin\alpha}{\sqrt{n_0^2 - sin^2\alpha}}; R^{*2} = \frac{RS\lambda}{\eta_0} - \frac{R\Delta}{\eta_0} + \frac{l^2}{16} \frac{sin^2\alpha}{(n_0^2 - sin^2\alpha)}$ (20)

Thus, as evident from (19), at the output of the CCL, (oo) and (eo) waves produce an interference pattern in the form of concentric rings with the radii R^{*2}, shifted along the x-axis at a distance "B", which is proportional to the angle α. Moreover, the distance between the adjacent maxima (rings) is given by the equality:

$$\Delta R = \frac{R\lambda}{2R^*\eta} \quad (21)$$

An analysis of the interference pattern with (oe) and (ee) beams superimposed is carried out in a similar way. With the details of calculation omitted, the final result can be provided as follows:

$$(X + B_1)^2 + Y^2 = R_1^{*2}, \quad (22)$$

where

$$B_1 = \frac{l}{4}\frac{sin\alpha}{\sqrt{\eta_0^2 - sin^2\alpha}}$$

$$R_1^* = \frac{RS\lambda}{\eta_1}\frac{R\Delta}{\eta_1} + \frac{l^2}{16}\frac{sin^2\alpha}{(n_e^2 - sin^2\alpha)}$$

$$\eta_1 = \eta_0^{||} - \eta_0^{|}; \quad \eta_0^{||} = \sqrt{1 - \frac{sin^2\alpha}{n_e^2}} - \frac{n_0}{n_e}\sqrt{1 - \frac{sin^2\alpha}{n_e^2}}$$

$$\eta_0^{|} = \sqrt{1 - \frac{sin^2\alpha}{n_e^2}} - \frac{n_0}{n_e}\sqrt{1 - \frac{sin^2\alpha}{n_e^2}}$$

R – the radius of curvature of the spherical surface; ℓ – the CCL thickness.

As seen from (22), at the output of the CCL, (eo) and (ee) waves also give an interference pattern in the form of concentric rings that are shifted along the x-axis at a distance "B", and the distance between the adjacent maxima (rings):

$$\Delta R = \frac{R\lambda}{2\eta_1 R_1^*} \quad (23)$$

With the help of the formula (23) and the knowledge of the radius of the first ring, one can determine the radii of the subsequent rings.

In all calculations, for simplicity, it was assumed that the interference pattern is observed in the vicinity of the rear (z=l) face of the CCL. It is easy to recount all the results for the screen located at any distance from this face. When the screen is moved, only the scale of the interference pattern is changed. For relevant assessment, it is necessary to know the wave vectors of all four beams. It is easy to show (from the boundary conditions at z=l) that these vectors can be written as:

$$\overrightarrow{K_3^{00}} = \{sin\alpha; o; cos\alpha\}$$

$$\overrightarrow{K_3^{oe}} = \left\{sin\alpha + \eta_0^{|}n_e\frac{ltg\alpha_1 + dcosa}{R};\ \eta_0^{|}n_0\frac{desin\varphi}{R};\ cos\alpha - \eta_0^{|}n_e g_1^{|}\right\}$$

$$\overrightarrow{K_3^{eo}} = \left\{ sin\alpha + \eta_0 n_e \frac{ltg\theta + decos\varphi_e}{R};\ \eta_0 n_0 \frac{desin\varphi_e}{R};\ cos\alpha - \eta_0 n_e g^{|} \right\} \quad (24)$$

$$\overrightarrow{K_3^{ee}} = \left\{ sin\alpha + \eta_0^{||} n_e \frac{ltg\theta + decos\varphi_e}{R};\ \eta_0^{||} n_e \frac{desin\varphi_e}{R};\ cos\alpha - \eta_0^{||} n_e g_2^{|} \right\}$$

,

where

$$g_1^{|} = \frac{sin\alpha}{\sqrt{n_e^2 - sin^2\alpha}} \frac{ltg\alpha_1 + dcos\varphi}{R}; \qquad g_2^{|} = \frac{sin\alpha}{\sqrt{n_e^2 - sin^2\alpha}} \frac{decos\varphi_e + ltg\theta}{R}$$

With the use of (24), find the expressions for the radii of the rings of interference patterns between (oo) and (eo) waves, and between (ee) and (oe) waves, respectively on the screen, at a distance L from the rear (z=l) face of the CCL.

Note that at normal incidence of the light beam on the input face of the CCL, the expressions (24) pass, as it should be, into the expressions (12), (14) and (15) at ψ=0.

In the case of divergent beams, the form of the interference pattern is complicated. The calculation of the beam path in this case, even in paraxial approximation, results in cumbersome expressions complicated for the physical analysis. However, if this approximation is supplemented by one more small parameter, it is possible to quite accurately describe the observed features of interference.

Represent the field at the output of the CCL in the form of partial plane waves, each of which corresponding to the unit wave vector of the form.

$$\overrightarrow{K_0} = (\alpha_0 cos\varphi_0;\ \alpha_0 sin\varphi_0; 1), \quad (25)$$

where α_0 is the angle between $\overrightarrow{K_0}$ and the z-axis, which determines a narrow cone of plane waves: φ and the d-radius vector, drawn from the origin of coordinates z=0 to the point M_1^e, and localizes a separate partial beam in the cone.

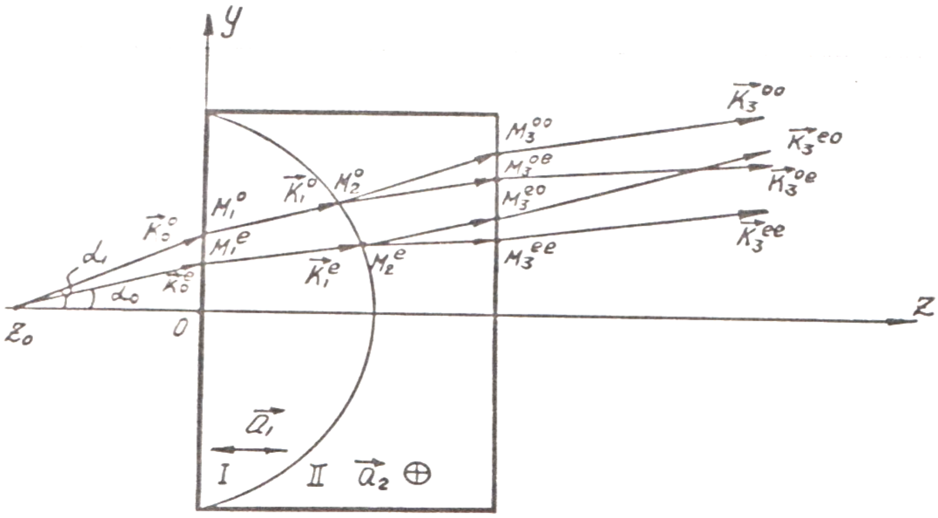


Figure 3. Scheme of the propagation of polarized rays through the CCL in the divergent laser beam (“without-analyzer” interference mode)

Assume that partial waves in the divergent beam are circularly polarized. In this case, when they are refracted in the plane z=0, there appear both o- and e-waves. Consider separately each of these waves with orthogonal polarization, experiencing transformation on the spherical surface of the CCL section.

Case A: transformation of the e-wave into (eo) and (ee) waves

The unit wave vectors of the refracted (eo) waves of the beam within the accuracy of α^2 and δ^2 are given by the ratio:

$$\overrightarrow{K_1^e} = \overrightarrow{K_2^{eo}} = \left\{\frac{\alpha_0 cos\varphi_0}{n_e};\ \frac{\alpha_0 sin\varphi_0}{n_e}; 1\right\} \tag{26}$$

The beam vector $\vec{S}$ within the accuracy of δ^2 is associated with the wave vector $\vec{K}$ by the ratio (26):

$$\vec{S} = \delta(\vec{K}\vec{a})\vec{a} + (1 - \delta\ (\vec{K}\vec{a})^2)\vec{K} \tag{27}$$

Hence, for the e-wave in the field I of the CCL, we have:

$$\overrightarrow{S_1} = \left\{\frac{\alpha_0 cos\varphi_0}{n_e};\ \frac{\alpha_0 sin\varphi_0}{n_e}; 1 + \delta\right\} \tag{28}$$

The coordinates of the input point M_1^e of partial (ee) and (eo) beams in the CCL and the points of their intersection with the inner spherical section boundary M_2^e are located on the direction vectors (25) and (28) of the beam path and are given by the ratios:

$$x_1^e = \alpha_0 z_0 cos\varphi_0;\ y_1^e = \alpha_0 z_0 sin\varphi_0;\ z_1^e = 0$$
$$x_2^e = \left[z_0 + \frac{R-\sigma}{n_e}(1-\delta)\right]\alpha_0 cos\varphi_0;$$
$$y_2^e = \left[z_0 + \frac{R-\sigma}{n_e}(1-\delta)\right]\alpha_0 sin\varphi_0; z_2^e = R-\sigma \qquad (29)$$

On the spherical boundary of the CCL section, e-waves are transformed into (ee) and (eo) waves with orthogonal polarizations. It is somewhat more complicated to find their trajectories in the field II of the CCL. However, the calculation is facilitated by the fact that the research provides a number of formulas for determining the wave vectors of the refracted waves in the case when the direction of the normal in the place of the beam's intersection of the boundary is known.

According

$$\overrightarrow{K_2} = \frac{\delta}{2}\frac{(\overrightarrow{\kappa_1 a_2})^2 - (\overrightarrow{\kappa_1 a_1})^2}{\overrightarrow{n_1 k_1}}\overrightarrow{n_1} + [1 - \delta((\overrightarrow{\kappa_1 a_2})^2 - (\overrightarrow{\kappa_1 a_1})^2)]\overrightarrow{k_1}\text{to} \qquad (30),$$

we have:

The normal to the spherical section boundary at the point M_2^e found from the equation of the spherical surface (28) and (29) has the form:

$$\overrightarrow{n_1^e} = \left\{\frac{\left[z_0 + \frac{R-\sigma}{n_e}(1-\delta)\right]}{R}\alpha_0 cos\varphi_0;\ \frac{\left[z_0 + \frac{R-\sigma}{n_e}(1-\delta)\right]}{R}\alpha_0 sin\varphi_0; 1\right\} \qquad (31)$$

The formulation (30) and (31) makes it possible to easily find the vector $\overrightarrow{K_2^{ee}}$:

$$\overrightarrow{K_2^{ee}} = \left\{\left[1 - \frac{\delta}{2}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R}\right)\right]\frac{\alpha_0 cos\varphi_0}{n_e};\ \left[1 - \frac{\delta}{2}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R}\right)\right]\frac{\alpha_0 sin\varphi_0}{n_e}; 1\right\} \qquad (32)$$

Find the beam vector $\overrightarrow{S_2}$ from (31):

$$\overrightarrow{s_2} = \left\{\left[1 - \frac{\delta}{2}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R} - 1\right)\right]\frac{\alpha_0 cos\varphi_0}{n_e};\ \left[1 - \frac{\delta}{2}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R} + 1\right)\right]\frac{\alpha_0 sin\varphi_0}{n_e}; 1\right\} \qquad (33)$$

The vectors (32) and (33) make it possible to determine the coordinates of the point M_3^{eo} and M_3^{ee}:

$$M_3^{eo} = \left\{\left[z_0 + 2\frac{R-\sigma}{n_e}\left(1 - \frac{\delta}{2}\right)\right]\alpha_0 cos\varphi_0; \left[z_0 + 2\frac{R-\sigma}{n_e}\left(1 - \frac{\delta}{2}\right)\right]\alpha_0 sin\varphi_0; 2(R-\sigma)\right\} \qquad (34)$$

$$M_3^{ee} = \left\{\left[z_0 + 2\frac{R-\sigma}{n_e} - \frac{\delta}{2}\frac{R-\sigma}{n_e}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R}\right)\right]\alpha_0 cos\varphi_0; \left[z_0 + 2\frac{R-\sigma}{n_e} - \frac{\delta}{2}\frac{R-\sigma}{n_e}\left(\frac{z_e}{R}n_e - \frac{\sigma}{R} + +2\right)\right]\alpha_0 sin\varphi_0; 2(R-\sigma)\right\} \qquad (35)$$

Case B: transformation of the o-wave into (oo) and (oe) waves

The calculation method of the trajectory of (oo) and (oe) beams is the same as above. We consider here only the final results of calculation:

$$\overrightarrow{K_2^{0}} = \overrightarrow{K_2^{00}} = \left\{\frac{\alpha_1 cos\varphi_1}{n_0};\ \frac{\alpha_1 cos\varphi_1}{n_0};1\right\} \quad (36)$$

$$\overrightarrow{K_2^{oe}} = \left\{\left[1-\frac{\delta}{2}\left(\frac{z_e}{R}n_e-\frac{\sigma}{R}+1\right)\right]\frac{\alpha_1 cos\varphi_1}{n_e};\quad \left[1-\frac{\delta}{2}\left(\frac{z_e}{R}n_e-\frac{\sigma}{R}+1\right)\right]\frac{\alpha_1 sin\varphi_1}{n_e};1\right\} \quad (37)$$

$$M_1^0 = \{\alpha_1 z_0 cos\varphi_1;\ \alpha_1 z_0 sin\varphi_1; 0\}$$

$$M_2^0 = \left\{\left[z_0+\frac{R-\sigma}{n_e}\left(1-\frac{\delta}{2}\right)\right]\alpha_1 cos\varphi_1;\left[z_0+\frac{R-\sigma}{n_e}\left(1-\frac{\delta}{2}\right)\right]\alpha_1 sin\varphi_1;(R-\sigma)\right\} \quad (38)$$

$$M_3^{00} = \left\{\begin{matrix}\left[z_0+2\dfrac{R-\sigma}{n_e}\left(1-\dfrac{\delta}{2}\right)\right]\alpha_1 cos\varphi_1; \\ \left[z_0+2\dfrac{R-\sigma}{n_e}\left(1-\dfrac{\delta}{2}\right)\right]\alpha_1 sin\varphi_1;2(R-\sigma)\end{matrix}\right\}$$

$$M_3^{oe} = \left\{\left[z_0+2\frac{R-\sigma}{n_e}-\frac{\delta}{2}\frac{R-\sigma}{n_e}\left(\frac{z_e}{R}n_e-\frac{\sigma}{R}\right)\right]\alpha_1 cos\varphi_1;\left[z_0+2\frac{R-\sigma}{n_e}-\frac{\delta}{2}\frac{R-\sigma}{n_e}\left(\frac{z_e}{R}n_e--\frac{\sigma}{R}\right)\right]\alpha_1 sin\varphi_1;2(R-\sigma)\right\} \quad (39)$$

Consider now the interference pattern at the output of the CCL. The electric field intensity of the light wave at the output of the CCL, according to Figure 4, can be written as follows:

$$\overrightarrow{E_1} = \overrightarrow{E_1} + \overrightarrow{E_0} = \overrightarrow{e_R}\mathrm{E}cos\varphi e^{i\overrightarrow{k_2^e}\vec{r}} + \overrightarrow{e_\varphi}\mathrm{E}sin\varphi e^{i\overrightarrow{k_2^0}\vec{r}+i\Delta l}, \quad (40)$$

where Δl is the phase shift due to the path difference of o- and e-waves, $\overrightarrow{e_\varphi}$ is the unit vector in the directions R and φ, and E is the field intensity of the incident wave.

If we go to the Cartesian coordinates, then:

$$\overrightarrow{e_R} = \overrightarrow{e_x}\,\mathrm{Cos}\varphi + \overrightarrow{e_y}sin\varphi;\ \overrightarrow{e_y} = \overrightarrow{e_x}\,\mathrm{sin}\varphi + \overrightarrow{e_y}con\varphi;, \quad (41)$$

where $\overrightarrow{e_x}, \overrightarrow{e_y}$ are the unit vectors of the x- and y-axes.

By substituting (40) into (41), we obtain:

$$\overrightarrow{E_1} = \overrightarrow{e_x}\left(cos^2\varphi e^{i\overrightarrow{k_2^e}\vec{r}} + sin^2\varphi e^{i\overrightarrow{k_2^0}\vec{r}+i\Delta l}\right)E^2 + $$
$$+\overrightarrow{e_y}\left(sin\varphi cos\varphi e^{i\overrightarrow{k_2^e}\vec{r}} - sin\varphi cos\varphi e^{i\overrightarrow{k_2^0}\vec{r}+i\Delta l}\right)E^2 \quad (42)$$

According to Figure 3, the electric field intensity of (ee) and (oe) waves on the rear face of the CCL is oriented toward the x-axis, and, according to (42), is equal to:

$$\overrightarrow{E_{IX}} = \overrightarrow{e_x} E e^{i\overrightarrow{k_2^{ee}}\vec{r}} \left(cos^2\varphi + sin^2\varphi e^{i\left[\left(\overrightarrow{k_2^{oe}} - \overrightarrow{k_2^{ee}}\right)\vec{r} + \Delta l\right]} \right) \quad (43)$$

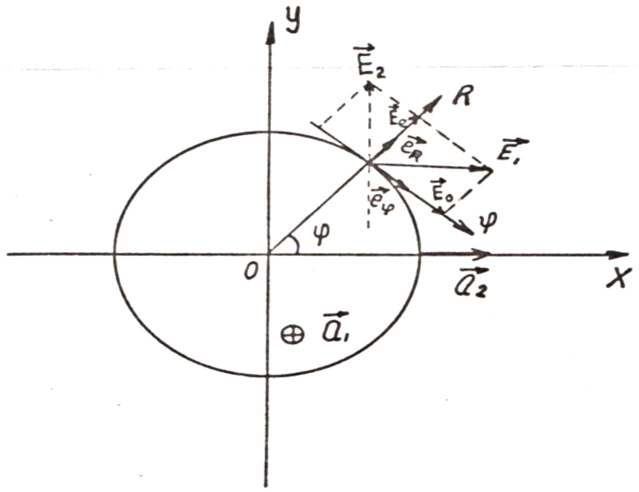


Figure 4. Polarization of waves at the output of the CCL in the divergent laser beam ("without-analyzer" interference mode)

The intensity of the full field can be written as follows:

$$I \sim |E_{IX}|^2 = |E|^2 \left| cos^2\varphi + sin^2\varphi e^{i\left[\left(\overrightarrow{k_2^{oe}} - \overrightarrow{k_2^{ee}}\right)\vec{r} + \Delta l\right]} \right|^2 \quad (44)$$

Hence, we obtain:

$$I \sim |E|^2 \left[1 - sin^2 2\varphi sin^2 \frac{\Delta L_1}{2} \right], \quad (45)$$

where

$$\Delta L_1 = \left(\overrightarrow{k_2^{oe}} - \overrightarrow{k_2^{ee}} \right) \vec{r} + \Delta l$$

The expression (45) shows that in the area of the angles $\varphi = 0$ and $\varphi = \frac{\pi}{2}$ there appears a bright field.

At $\varphi = \frac{\pi}{4}$, following from (45), we obtain

$$I \sim |E|^2 \left[1 - sin^2 \frac{\Delta L_1}{2} \right] \quad (46)$$

The condition of interference maxima can be written as follows:

$$\frac{\Delta L_1}{2} = \pi s, \quad (47)$$

where S=0; ± 1; ± 2; ± 3;

The vector of the electric field intensity of (oo) and (eo) waves is oriented toward the y-axis. In accordance with (42), we have:

$$\vec{E}_{1y} = \vec{e}_y E\sin\varphi cos\varphi e^{i\vec{k}_2^{eo}\vec{r}}[1 - e^{i[(\overrightarrow{k_2^{eo}} - \vec{k}_2^{eo})\vec{r} + \Delta l']}], \quad (48)$$

where $\Delta L'$ is the phase shift due to the path difference of (oo) and (eo) waves.

Following from (48), the intensity of the field can be written as follows:

$$I \sim E^2 sin^2 2\varphi sin^2 \frac{\Delta L_1}{2}, \quad (49)$$

where $\Delta L_2 = (\vec{k}_2^{e0} - \vec{k}_2^{e0})\vec{r} + \Delta l'$

As seen from (50), in the area of the angles φ=0 and $\varphi = \frac{\pi}{2}$ there appears a dark field.

At $\varphi = \frac{\pi}{4}$ the condition of interference maxima can be written as:

$$\frac{\Delta L_2}{2} = (S + \frac{1}{2})\pi, \quad (50)$$

where s=0; $\pm$1; $\pm$2;

To determine the locus of the interference pattern maxima and minima points, it is necessary to find the functional dependence of the angles $\varphi_0, \varphi_1, \alpha_0, \alpha_1$. The fact is that this point of the screen, where the interference pattern is observed, gets the partial beams having different CCL input points. The condition of overlapping these beams on the screen provides the necessary dependence.

Thus, for (ee) and (oe) waves we have:

$$\tan\varphi_0 = \frac{y_3^{ee}}{x_3^{oe}}\left[1 + \frac{\delta}{L}\frac{R - \sigma}{n_e}\right]$$

$$\tan\varphi_1 = \frac{y_3^{oe}}{x_3^{oe}}\left[1 + \frac{\delta}{L}\frac{R - \sigma}{n_e}\right]$$

$$\alpha_0 = \frac{x_3^{oe}}{L}\sqrt{1 + \tan^2\varphi_0}\left[1 + \frac{\delta}{2L}\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e - \frac{\sigma}{R}\right)\right] \quad (51)$$

$$\alpha_1 = \frac{x_3^{oe}}{L}\sqrt{1 + \tan^2\varphi_1}\left[1 + \frac{\delta}{2L}\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e - \frac{\sigma}{R}\right)\right],$$

where L=$Z_0 + 2\frac{R-\sigma}{n_e}$

Taking into account the expressions (51), from the formulas (25) and (30) we obtain

$$\vec{k}_2^{ee} = \left\{ \frac{x_3^{ee}}{Ln_e}\left[1+\frac{\delta}{2L}\left[\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}\right)-L\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}\right)\right]\right]; \right. \tag{52}$$

$$\left. \frac{y_3^{ee}}{Ln_e}\left[1+\frac{\delta}{2L}\left[\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}+2\right)-L\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}\right)\right]\right]:1\right\}$$

$$\vec{k}_2^{oe} = \left\{ \frac{x_3^{oe}}{Ln_e}\left[1+\frac{\delta}{2L}\left[\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}\right)-L\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}+1\right)\right]\right]; \right. \tag{53}$$

$$\left. \frac{y_3^{oe}}{Ln_e}\left[1+\frac{\delta}{2L}\left[\frac{R-\sigma}{n_e}\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}+2\right)-L\left(\frac{Z_0}{R}n_e-\frac{\sigma}{R}+1\right)\right]\right]:1\right\}$$

The conditions of interference maxima (40) can be written as follows:

$$\left(\vec{k}_{2x}^{oe}-\vec{k}_{2x}^{ee}\right)x+\left(\vec{k}_{2y}^{oe}-\vec{k}_{2y}^{ee}\right)y+\Delta= s\lambda, \tag{54}$$

where Δ is the path difference between (oe) and (ee) waves at the output of the CCL. The optical paths of (oo), (oe), (ee) and (oe) waves in the CCL within the accuracy of up to α^2 are equal:

$Z=2(R-\sigma)$

Therefore, the path difference Δ at the output of the CCL is zero ($\Delta= 0$). Thus, by substituting (52) and (53) into (54), obtain

$$(x^2+y^2)= s\lambda\frac{L}{\delta} \tag{55}$$

Thus, (ee) and (oe) waves at the output of the CCL at $\varphi=\frac{\pi}{4}$ produce an interference pattern in the form of rings with the radii:

$$R=\sqrt{s\lambda\frac{L}{\delta}},\quad S=1,2,3.......$$

For (oo) and (eo) waves, we have:

$$\tan\varphi_0=\frac{y_3^{eo}}{x_3^{eo}};\quad \tan\varphi_1=\frac{y_3^{oo}}{x_3^{oo}};$$

$$\alpha_0=\frac{x_3^{eo}}{L}\sqrt{1+\tan^2\varphi_0}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right];$$

$$\alpha_1=\frac{x_3^{oo}}{L}\sqrt{1+\tan^2\varphi_1}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right] \tag{56}$$

By substituting the expression (49) into (26) and (36), obtain:

$$\vec{k}_2^{oe} = \left\{\frac{x_3^{eo}}{Ln_e}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right];\frac{y_3^{eo}}{Ln_e}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right];1\right\} \quad (57)$$

$$\vec{k}_2^{oo} = \left\{\frac{x_3^{oo}}{Ln_e}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right];\frac{y_3^{oo}}{Ln_o}\left[1+\frac{\delta}{L}\frac{R-\sigma}{n_e}\right];1\right\}$$

Taking into account (58), from the condition of the interference maximum (50) we have

$$(x^2+y^2) = (2s+1)\frac{\lambda}{2}\frac{L}{\delta} \quad (58)$$

As can be seen from (59), (oo) and (eo) waves as well as (ee) and (oe) waves at $\varphi=\frac{\pi}{4}$ at the output of the CCL will produce an interference pattern in the form of rings with the radii:

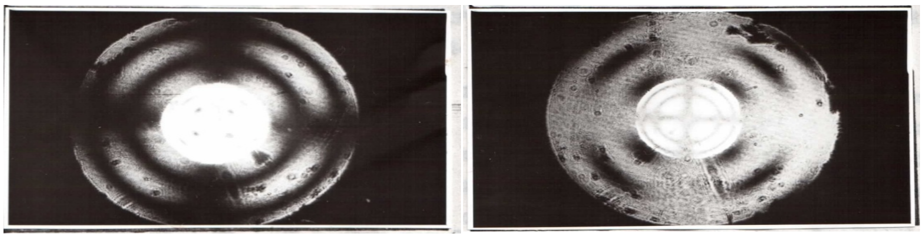
$$R = \sqrt{2S+1}\frac{\lambda}{2}\frac{L}{S}, \quad (S=0;1;2;)$$

Thus, in the CCL, two interference patterns are formed in the divergent (convergent) laser beam (without the use of the analyzer): one pattern in the form of a light cross, intersected by rings, and the other – in the form of a dark cross with rings (Figure 5a, 5b).

When viewed on the screen at a distance L_1 from the rear face of the CCL, the expressions (55) and (58) can be written in the form:

$$(x^2+y^2) = \frac{S\lambda}{\delta}\frac{(L+L_1)^2}{L} \quad (59)$$

$$(x^2+y^2) = (2s+1)\frac{\lambda}{2\delta}\frac{(L+L_1)^2}{L} \quad (60)$$



a. b.

Figure 5. Interference bitmaps generated by the CCL in the convergent laser beam ("without-analyzer" interference mode); a. $\vec{E} \perp \vec{a_1}$ /a light cross in the center/ b. $\vec{E} \parallel \vec{a_1}$ /a dark cross in the center/

When turning the CCL around the y-axis by 180º, the above-mentioned interference patterns are preserved, but only when using the analyzer.

Conclusion

The study of a crystalline compound lens helped to determine the basic properties related to the transformation of spherical laser radiation and the formation of different polarized beams at its output as well as the condition for the emergence of interference between them. It also helped to obtain the expressions convenient for engineering calculations and to determine the conditions for the formation of an interference pattern of the CCL with and without the use of the analyzer. The authors also examined the prospects of using interference patterns formed by the CCL in various laser measurement devices, including polarized interferometers.

The paper presented a number of expressions describing the propagation of laser radiation through crystalline lenses and determining the transformation of different (ordinary and extraordinary) waves on the spherical surface of a crystalline lens. The expressions obtained with great accuracy coincide with the experimental data.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Seitbek Bekenovich Kyanishbayev is a Professor of Chemistry, Biology and Geography Department at Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Abilhan Umbetovich Umbetov is an Associate Professor of Mathematics and Physics Department at Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Aisaule Esengazinovna Duisebekova is a PhD of Preschool and Primary Education Department at Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Makhabbat Zhaksylykovna Umbetova is a Master and Senior Lecturer of Preschool and Primary Education Department at Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Kamar Serikbaevna Schongalova is a Master and Senior Lecturer Professor of Informatics Department Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Zhanar Edresovna Akhatova is a Senior Lecturer of Informatics Department Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Perizat Seithankyzy Azhibekova is a Master and Lecturer of Informatics Department Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

Aigerim Sharipovna Sokabayeva is a Master and Lecturer of Mathematics and Physics Department at Arkalyk State Pedagogical Institute named after Y. Altynsarin, Arkalyk, Kazakhstan.

References

Born, M. & Wolf, E. (1999). *Principles of optics*. Cambridge: Cambridge University Press.

Collier, R.J., Burkhardt, C.B. & Lin, L.H. (1971). *Optical holography*. New York: Academic Press.

Dreytsen, G.V., Ostrovoskin, Yu.P. & Shedeva, E.P. (1972). An interference method for studying the degree of spatial coherence. *Optics and Spectroscopy,* 32, 367-378.

Fedorov, F.I. & Filippov, V.V. (1968). Light reflection at the boundary of a uniaxial crystal with the isotropic medium. *Applied Spectroscopy, 9*, 1031-1045.

Fedorov, F.I. & Filippov, V.V. (1971a). Amplitude ratios for light waves at the boundary of a uniaxial crystal with the isotropic medium. *Optics and Spectroscopy, 30,* 318-322.

Fedorov, F.I. & Filippov, V.V. (1976). *Light reflection and refraction by transparent crystals*. Minsk: Nauka.

Fedorov, F.I. & Konstantinova, A.F. (1962). The propagation of light through plates of uniaxial optical active crystals of axial classes. *Optics and Spectroscopy, 12,* 407-418.

Fedorov, F.I. & Kotyash, T.L. (1962) The propagation of light through plates of a transparent uniaxial crystal. *Optics and Spectroscopy*, *12,* 298-302.

Fedorov, F.I., Filippov, V.V. (1971b). On the reflection and refraction of extraordinary beams in uniaxial crystals. *Crystallography, 16*, 36-46.

Galperin, D.Yu. (1959). *Research on geometrical optics (*Doctoral dissertation). Leningrad: The Vavilov State Optical Institute.

Javan, A., Bennett Jr. W.R. & Herriot, D.R. (1961).Population inversion and continuous optical maser oscillator in a gas discharge containing a He–Ne mixture. *Physical Review Letters, 6(3),* 106-110.

Kulcke, W., Kosanke, K., Max, E., Fleisher, H. & Harris, T. J. (1965). Convergent beam digital light deflector. In: Tippett, J. T., Berkowitz, D. A., Clapp, L. C., Koester, C.J. & Vanderburgh, Jr. A. *Optical and Electro-Optical Information Processing*. MIT Press.

Landsberg, F.S. (1976). *Optics*. Moscow: Nauka.

Ostrovskiy, Yu.I. (1968). Interference resolvometer. USSR Patent No. 207018. *Bulletin of Inventions, 9,* 25-33.

Smith, P.W. (1972). Mode selection in lasers. *Proceedings of the IEEE, 60(4),* 422–440.

Soroko, L.M. (1971). *Basics of coherent optics and holography*. Moscow: Nauka.

Yu, F.T. (1973). *Introduction to diffraction, information processing and holography.* London: The M.I.T Press.