LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
2017, VOL. 12, NO. 8, 1709-1717

Ghanaian Senior High School Students' Error in Learning of Trigonometry

Farouq Sessah Mensah[a]

[a]University of Cape Coast, Department of Mathematics and ICT Education, GHANA

ABSTRACT

The purpose of the study was to explore students' error in learning trigonometry. The samples were 100 students who were final year students preparing to write the 2017 West African Senior School Certificate Examination (WASSCE) in the Central region of Ghana. The Mathematics Achievement Tests (MAT) and Trigonometrical Diagnostic Test (TDT) were used as the instruments of this study that included two components: the use of formula and right-angled method. Diagnostic interview was also used to identify the level at which students' errors occur in solving problems involving trigonometry. The type of errors is based on Newman Error Hierarchy Model that includes reading type error, comprehension, transformation, process skill, encoding error and carelessness. The findings of the study showed that most students make error in process skill and transformation irrespective of the method used in solving trigonometry problems. There was no error found in students' reading skills. The number of students who made encoding error and carelessness was relatively low. The students' error in solving trigonometry problems was due to their weaknesses in basic arithmetical operations. The implication of the study was highlighted and recommendations were made based on the findings.

KEYWORDS
Trigonometry, process skill error, transformation error, Comprehension error

ARTICLE HISTORY
Received 13 April 2017
Revised 27 August 2017
Accepted 4 September 2017

Introduction

Problems associated with the Ghanaian educational system are very diverse in nature, including the problems associated with mathematics education. Mathematics is required in our daily lives, regardless of our educational background or social status. The benefit of mathematics is not only limited to knowledge in computation, but more important, when each individual can master mathematics well, then their pattern of thinking is more rational and critical. The principles of mathematics enable people to see the problems as facts not as fiction (Hudoyo, 1998).

Trigonometry is one of the mathematics content that very few students like and likely to succeed at, and which most students hate and struggle with.

CORRESPONDENCE Farouq Sessah Mensah Farouq.mensah@stu.ucc.edu.gh

Trigonometry is an area of study in mathematics that student believe to be particularly difficult and abstract compared with the other content areas in mathematics. Trigonometry is often introduced early in Junior High School (JHS) with most textbooks traditionally starting with naming sides of the right-angled triangles. Students need to see and understand why their learning of trigonometry matters (Gur, 2009).

Error in solving mathematical problems often occurs either in writing, or orally (Zakaria, 2010). During the process of teaching and learning mathematics, students are faced with many obstacles because problem solving in mathematics is a skill that is very complicated. Many studies concerned with mathematics education explain that students have misconceptions and make errors, and these situations grow out of learning complexities (Ryan, 1992). In recent time times, few researchers have mentioned students' misconceptions, errors and the learning complexities about trigonometry (Orhun, 2015).

N. Orhun (2015) studied the difficulties faced by students in solving problems in trigonometry. N. Orhun (2015) found that the students did not develop the concepts of trigonometry certainly and that they made some mistakes.

Errors in Trigonometry

Trigonometry, is a branch of mathematics that deals with the relationships of sides and angles in triangles. It forms an important background for the solution of problems in many disciplines (Orhun, 2010). Trigonometry is frequently used in mathematical explanations and definitions of new ideas and concepts. For example, trigonometric ratios are used to describe the relationship between angles and their sides in right-angled triangles. Studies have shown that many students have not developed clear concepts in trigonometry and that some of them use algebraic notation informally (Maharaj, 2008).

N. Orhun (2010) states that students registered for calculus in their first year at a university perform badly in the operations of trigonometric expressions, namely addition, subtraction, multiplication and division. For example, students demonstrate difficulties in the multiplication of $Sin\,x \times Sin\,x$. N. Orhun (2010) argues that this may be due to the fact that there may not be much emphasis in the learning of addition, subtraction, multiplication and division of trigonometric functions in the SHS curriculum. This is evident from Table 1 which shows the content coverage for trigonometry at the SHS level as prescribed by Ministry of Education, Science and Sports. (2010). N. Orhun (2010) suggests that, "In order for teachers to account for students' systematic errors from a constructivist perspective, analysing the procedures is not sufficient, since teachers should analyse students' current schemas and how they interact with each other according to instruction and experience".

M.E. Skane and A.O. Graeber (1993) claim that some errors displayed by students in the content of algebra, logarithms, exponents and trigonometry are attributed to the distributive law. They further suggest that traditional instruction is not a sufficient strategy to remediate distributive law errors for some students. This applies in compound angle formulae such as $Cos(x+y) = CosxCosy - SinxSiny$. The use of distributive property in an algebraic expression familiarises students with $x(a+b) = ax + ab$. Researchers such as P.W.

Thompson, B. Byerley & N. Hatfield (2013) suggest the use of a conceptual approach to teach trigonometry as a better option compared to the traditional approach. The traditional approach commonly known as the procedural approach may be defined as teacher-led with direct instruction of rules or procedures for solving problems (Peal, 2010). The opposite end of the instructional spectrum from the procedural approach is the conceptual approach. Conceptual-based instruction seeks to provide reasons why these algorithms and formulae work (Peal, 2010). In the conceptual approach, the emphasis is on the students' learning important concepts of mathematical connections, relationships and applications. The rise of the conceptual approach in mathematics instruction is consistent with a constructivist approach to education (Peal, 2010).

Table 1. Trigonometry in SHS Curriculum in Ghana

Core Mathematics		Elective Mathematics
SHS 2	**SHS 3**	**SHS 2**
The trigonometric ratios of $30°, 45°$ and $60°$.	Graphs of trigonometric functions	Basic trigonometric ratios and their reciprocals
The use of calculators to read sine, cosine and tangent of angles between $0°$ and $360°$	Trigonometric equations	Angles in radians
Inverse of trigonometric ratios		Sine and Cosine rules
Angles of elevation and depression		Application of sine and cosine rules to bearings
Application of trigonometric ratios		Compound angles
		Multiple angles (up to 3A)
		Graphs of trigonometric functions
		Solving trigonometric equations (up to quadratic)
		Maximum and minimum points of trigonometric functions

Source: Ministry of Education, Science and Sports. (2010).

Conceptual Framework

The conceptual framework that is used in this study is based on Newman Error Hierarchical Model. The model of error investigation proposed by M.A. Newman (1997) has proved to be a reliable model for mathematics teachers. The framework has six types of errors: reading error, comprehension, transformation, process skill, encoding error and carelessness. The Newman Error Hierarchical Model is suitable to be used in identifying students' error in mathematics. This model has the hierarchy that classifies types of error based on the problem solving level done by students. According to M.A. Clements and N.F. Ellerton (1996), M.A. Newman (1997) used the word "hierarchy" because she reasoned that failure at any level of the sequence prevents students from obtaining satisfactory solutions. N. Prakitipong and S. Nakamura (2006) pointed out, in the process of problem solving there are two kinds of obstacle that hinder students from arriving at correct answers:

a. Problems in linguistics fluency and conceptual understanding that correspond with level of simple reading and understanding meaning of problems, and

b. Problems in mathematical processing that consists of transformation, process skills, and encoding answers.

This classification implies that the students have to interpret the meaning of the question before they proceed to mathematical processing to obtain appropriate answer. The outcome of the written students' work in their test will be evaluated according to the type of error in Newman Error Hierarchical Model based on the first breakdown point. The conceptual framework is shown in Figure 1.

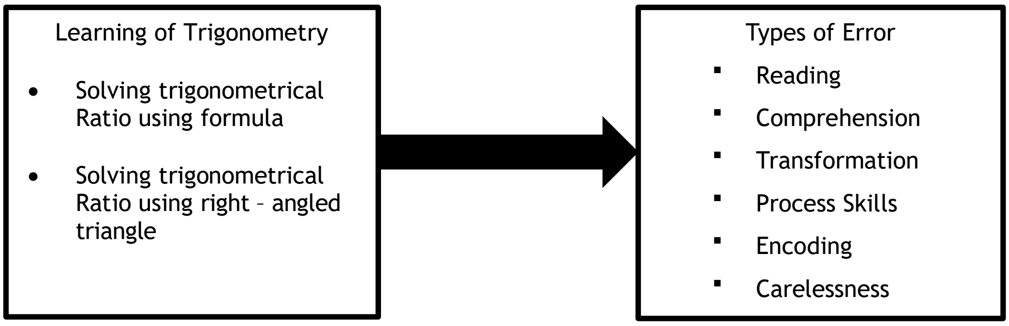

Figure 1. Conceptual Framework of the study
Source: Developed by Researcher

Purpose of the Study

The purpose of this study was to analyze students' error in learning trigonometry which focused on which focused on the manipulation of trigonometrical ratios using formula and the right-angled triangle.

Research Question

Two research questions were posed for this study:

1. What are students' errors in solving trigonometrical ratio using formulae?

2. What are students' errors in solving trigonometrical ratio using right – angled triangle?

Methodology

This study is located within the interpretative qualitative research paradigm. Qualitative research is an exploratory approach, which emphasises the use of open – ended questions and probes, giving participants an opportunity to respond to their own words (Devetak, Glazar & Vogrinc, 2010).

Research Participant

The research participants of the study were the 100 students who were final year students preparing to write the 2017 West African Senior School Certificate Examination (WASSCE) in the Central region of Ghana. The justification of these students as respondents is the fact that, the curriculum indicates that, trigonometry must be taught in the second and third year of SHS.

Instrumentation

The study used a set of Trigonometric Diagnostic Test (TDT) (Table 2) questions designed by the researcher to identify type of error committed by students. Prior to the commencement of the study, the questions were subjected to face and Pearson's Product Moment content validated test respectively. The reliability coefficient as computed from Pearson's Product Moment correlation for Trigonometric Diagnostic test (TDT), (see Appendix A) was $r = 0.93$. The value suggest that the test question were reliable and as such would test what it was designed for. In addition, content and face validity of the instrument was carried out by the research who is a specialist in mathematics and other specialists in cognate field compared to the overall objective of the curriculum.

Prior to the investigation, the respondents were subject to general Mathematics Achievement Test (MAT). The primary objective of the MAT items was to categorize the students into high achievers (A) for those that scored 70.0% and above medium achievers (B) for those that scored $50.0 - 69\%$ and low achievers (C) for those who scored 49.0% or less (Table 3). The researcher also interviewed the respondent based on the modified procedure of Newman Error Hierarchical Model from the outcome of the written test (see Appendix B).

Table 2. Number of Items (TDT) According to Skills

Method of Solving Trigonometry Ratio	Skill	Item Number
Formulae	Solving trigonometrical ratio using formula	1 - 7
Right Angled – Triangle	Solving trigonometrical ratio using right angled - triangle	1 - 7

Source: Developed by Researcher

Table 3. Distribution of Students by Achievement Levels

Gender	A	B	C	Total	%
Male	16	20	10	46	46.0
Female	21	18	15	54	54.0
Total	**37**	**38**	**25**	**100**	**100.0**

Source: Field Data, 2017

Results

The respondent's profile according to gender and their achievement is represented in Table 3 above. As shown in Table 3, the students were grouped in three different category of achievement: low, medium and high. From Table 2, 16 male and 21 female students belonged to the high achievers category, 20 male and 18 female belonged to the medium achievers category and the low category of achievers had 10 males and 15 females.

Types of Error in Using Formulae in Solving Trigonometrical Ratio

From item 1 to 7, most type of error made by students using formulae in solving trigonometrically ratio were process skills errors followed by transformation errors. The details of the error type based on the items are shown in Table 4.

Table 4: Error in solving Trigonometry Ratio by Formulae

Item	Readi ng	Compreh ension	Transforma tion	Process Skills	Encoding	Careles sness
1	0	10	35	37	7	8
2	0	6	37	45	5	2
3	0	7	40	41	7	6
4	0	9	31	47	4	0
5	0	7	35	41	2	0
6	0	9	33	49	4	3
7	0	12	35	45	7	2
Total	**0**	**60**	**246**	**305**	**36**	**21**

Source: Field Data, 2017

Types of Error in Using Right Angled - Triangle in Solving Trigonometrical Ratio

For item 1 to 7, there was a total of 39 comprehension error, 263 transformation error, 324 process skill error, 52 encoding error and 14 carelessness. Most type of errors made by students were process skill errors. The details of the error made according to their type are shown in Table 5.

Table 5: Error in solving Trigonometry Ratio by Right Angled - Triangle

Item	Readi ng	Compreh ension	Transform ation	Process Skills	Encoding	Careles sness
1	0	12	39	39	6	6
2	0	8	32	46	9	4
3	0	19	43	53	4	0
4	0	7	36	44	2	2
5	0	9	39	41	5	0
6	0	12	31	53	3	2
7	0	16	43	48	3	0
Total	**0**	**83**	**263**	**324**	**32**	**14**

Source: Field Data, 2017

Discussion

The most frequent errors made by student in using formulae and right angled - triangle in solving trigonometrical ratio include process skills error, transformation errors and comprehension errors.

The error in process skill seems more pronounced in the use of right angled triangle than using formulae. This error may be due to the fact that students failed to understand and describe what is required by the questions. Most students did not manage to perform the operation especially when numerical values are involved. This results in failure to solve the problems. The results of the study is consistent with the findings of A. Norasiah (2002) in which problematic students failed to translate mathematical problems into mathematical form and also having problem in understanding the special terms in mathematics. This failure may be caused by lack of emphasis by teachers on understanding the language of trigonometry and the skills needed by the students. N. Orhun (2015) puts the buck of students' error in trigonometry at the teachers table and noted that when trigonometry is taught via teacher-active method with constant memorizing of the ready knowledge and repeating them will make the students learn trigonometry effectively. For example, emphasis on the trigonometrical acronym of SOH CAH TOA and their reciprocals as well as the foundational Pythagoras Theorem will go a long way in preparing the students for a logical and systematic learning of trigonometry. This in the view of the authors is probably why the students write the answers that are at variance with concepts and method learnt in class.

The error type in transformation occurred during computation process especially during multiplication. This takes place due to computation problem especially among low achievers. Most students make error at the process skill level especially in the manipulation of trig ratios using right angled - triangle. For example in item 3 the students were required to find $Cosec\ A$ given that $Tan\ A$ is 5/12, it was found that most students who got the item by the use of right-angled triangle method could not replicate same answer using the formula as a result of inappropriate placing and manipulation. The findings of the study also support the research of A. Norasiah (2002), that most low and average students face difficulty in performing trigonometrical operations.

Most comprehension errors occur when students do not understand how to approach a given trigonometrical problem from the concept. Students often misunderstand the demands of trigonometric question. This may be due to the lack of emphasis by teachers in teaching the simplification of concepts as they appear. It may also be due to rote learning on the part of the learner. Gur (2009) noted that errors committed by students in learning trigonometry may be useful for the teacher in evaluating his teaching so as to be able to correct the students as and when appropriate. Therefore, teachers must ensure that the teaching of mathematical concepts must be balanced with the arithmetic skills. The findings from the present study is consistent with previous studies (Delice, 2002; Intanku 2003; Weber, 2005; Brown, 2006, Zakaria, 2010).

Results of this study are in line with the research findings of S.T. Liew and S.W-H. Wan Muhammad (1991) who observed that emphasis on algorithmic skills without explanation on the concept or principle are the factors that cause difficulties in mathematics and by extension poor achievement in trigonometry.

Conclusion and Recommendation

The results of this study revealed that most errors committed by students irrespective of the method used were process skills errors and transformation errors. The study further shows that generally, students irrespective of different cognitive ability are susceptible to error in solving problems in trigonometry. Research on analysis of students' error in solving trigonometrical ratio will be of the most beneficial to SHS mathematics. The analysis of each student's problem will enable the teachers to plan their teaching effectively and meaningful. Based on the analysis, teachers will identify the roots of the problems in students' learning. As a result, the outcome of the research can be shared with their colleagues in order to solve the students' problem. This study will also help mathematics educators get information on the students' problem in mathematics learning. In order to counter students' problem, teachers should be well prepared and be able to conduct analysis on students learning. The findings of this study can be as a reference for teachers to find alternative ways in solving students' problem especially in solving trigonometry ratios.

Disclosure statement

The Authors reported that no competing financial interest.

Notes on contributors

Farouq Sessah Mensah - University of Cape Coast, Department of Mathematics and ICT Education, Cape Coast, Ghana

References

Brown, A.S. (2006). The trigonometric connection: students" understanding of sine and cosine. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, 1,* 228-236. Prague.

Clements, M.A. & Ellerton, N.F. (1996).The Newman procedure for analysing errors on written mathematical tasks. Retrieved January 12, 2017, from http://compasstech.com.au/ARNOLD/PAGES/newman.htm

Devetak, I, Glazar, S.A, & Vogrinc, J. (2010). The role of qualitative research in science education. *Eurasia Journal of Mathematics, Science and Technology Education, 6*(1), 77–84.

Gur, H. (2009). Trigonometry Learning. *New Horizons in Education,* 57(1), 67- 80.

Hudoyo, H. (1998). *Pembelajaran Matematika Menurut Pandangan Konstruktivistik*. [Learning mathematics through constructivist perspectives]. *Conference Paper Seminar Nasional Upaya Meningkatkan Peran Pendidikan Matematika dalam Menghadapi Era Globalisasi*. PPS IKIP Malang. Indonesia.

Intanku, S.S. (2003). Diagnosis for the type of error in differentiation. *Unpublished Master of Education Research Project*. Malaysia: Universiti Kebangsaan Malaysia

Liew, S.T. & Wan Muhamad Saridan Wan Hasan. (1991). Understanding and minimizing difficulty in learning mathematics *Berita Matematik,* 38, 22-29

Maharaj, A. (2008). Some insights from research for teaching and learning mathematics. *South African Journal of Education*, 28, 401–414.

Ministry of Education, Science and Sports. (2010). *Teaching syllabus for mathematics (Senior High School).* Accra: Curriculum Research and Development Division (CRDD).

Newman, M.A. (1977). An analysis of 6th grade pupils' errors on written mathematical task. Dlm. Clements, M.A. & Foyster, J. (Eds). *Research in Mathematical Education in Australia*: 239-258.

Norasiah, A. (2002). Error type diagnosis in learning simultaneous equation. *Unpublished Master of Education Research Project*, Universiti Kebangsaan Malaysia

Orhun, N. (2010). The gap between real numbers and trigonometric relations. *Quaderni di Ricerca in Didattica*, 20, 175–184.

Orhun, N. (2015) *Students' Mistakes And Misconceptions On Teaching Of Trigonometry*. 1st ed. Web. 14 Jan. 2015.

Peal, D. (2010). Approaches to teaching with mathematics. Connection, 1(3), 1–12.

Prakitipong, N. & Nakamura, S. (2006).Analysis of mathematics performance of Grade 5 students in Thailand using Newman procedure. *Journal of International Cooperation in Education, 9*(1), 111-122.

Ryan, J. (1992). Integrating computers into the teaching of calculus: differentiating student needs. In B Southwell, B Perry, & K Owens (Eds.), *Proceedings of the 13thAnnual Conference of the Mathematics Education Research Group of Australasia* (pp. 478–487). Richmond: University of Western Sydney.

Skane, M.E & Graeber, A.O. (1993). *A conceptual change model implemented with college students: distributive law misconceptions. Third International Conference on Misconceptions in Science and Mathematics*. New York: Ithaca.

Thompson, P.W., Byerley, B. & Hatfield, N. (2013). A conceptual approach to calculus made possible by technology. *Computers in the schools*, 30, 124–147.

Weber, K. (2005). Students" understanding of trigonometric functions. *Mathematics Education Research Journal. 17*(3), 91-112.

Zakaria, E. (2010) Analysis of Students" Error in Learning Quadratic Equations. *International Education studies. 3*(3) 105-110.

APPENDIX A **TRIGONOMETRIC DIAGNOSTIC TEST (TDT)**	**APPENDIX B** **INTERVIEW GUIDE**
1. Show that $Sin\,^2A + \,Cos^2A = 1$. 2. Give that $Sin\,A = 0.80$. Find $\tan A$. 3. Find $Cosec\,A$ when $tan\,A$ is $\frac{5}{12}$. 4. Given that $Cos\,60° = 0.05$, what is $Cosec\,30°$? 5. Find $Cot\,B$ when $Sec\,B = \frac{p}{q}$. 6. Simplify $(Sin\,A + Cos\,A)^2$ 7. Find $Tan\,A$ when $Cosec\,A = \frac{17}{5}$	1. Can you read the problem? (Reading) 2. What does the question demands of you? (Comprehension) 3. How do you solve the question? (Transformation) 4. Please show me the steps involved in solving the question. (Process Skills) 5. Kindly tell me your answer. (Encoding) 6. Do you see any error in your presentation? (Carelessness)