LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
e-ISSN: 1306-3065
2019, Vol. 14, No. 7, 435-444

Investigating Student's Geometric Thinking in Looking for the Linkages of Quadrilaterals (A Case Study on Students in The Formal Operation Stage)

Agustan Syamsuddin [1,2*]

[1] *Mathematics Education Department, Universitas Muhammadiyah Makassar, INDONESIA*
[2] *Magister of Elementary of Education Postgraduate Program, Universitas Muhammadiyah Makassar, INDONESIA*

* CORRESPONDENCE: ✉ agustan@unismuh.ac.id

ABSTRACT

The purpose of this paper is to investigate the ability of students in understanding the relationship of quadrilateral based on their geometric thinking skill. It can help the students in solving problem to get a correct solution. The participants of this paper involved two students eighth-grade from 2 secondary schools in the area of Kabupaten Bone, Makassar, Indonesia. The participants were chosen, based on Piaget's cognitive development, respectively one student at the early formal and late formal operation stage. To investigate students's geometric thinking in constructing their understanding about the linkage of quadrillaterals, the researcher conducted task-based interviews by drawing activities, identifying and stringing the relationship between quadrilaterals. The results of this research showed that the student at the early formal stage (PEFO) stringing 14 of 15 relationships and tends to use 3 attributes (position, size, shape of quadrilateral). Addition the student at the late formal stage (PLFO) stringing 7 of 15 relationships and tends to use 4 attributes (position, size, shape of quadrilateral and rotational symmetry). There are all participants' tendencies in drawing a quadrilateral always start drawing rectangel or square. It indicates that students are very familiar both of them. Therefore, it suggests that, in teaching, a teacher does not only start from a square or rectangle but also a teacher can start another quadrilateral such as parallelogram, rhombus, kite and trapezoid.

Keywords: geometric thinking, formal operation, quadrilateral

INTRODUCTION

Geometry occupies a special position in the secondary mathematics curriculum because there are many concepts contained. The concept is closely associated to other forms of objects that are often encountered by students in everyday life. Various opinions emerged that addresses geometry both definitions and chances to be taught in schools. Geometry learning must be allowed to the students' thinking skills will enhance the intellectual engagement of students.

Abdussakir (2002) stated that, basically, geometry has a better chance to be understood by students for ideas of geometry have been already known by students since before they are in the school, for example, line, area, and space. However, the expectation is different from the reality in real life where many studies showed that mastery of concept in mathematics, especially geometry achievement is still low (Husnaeni, 2006). It is supported by the results of Setiawan's research (Suyanto, 2005) stated that the fifth grade of elementary school students did not master the concepts and principles of geometry. While, in junior high school, Sunardi (2005)

Article History: Received 22 June 2019 ◆ Revised 19 July 2019 ◆ Accepted 19 July 2019

found that many students were wrong in solving the problems of parallel lines. Based on this findings, geometry is seen as part of the mathematics lessons given to students classified as difficult concept. Student's learning difficulties can not be separated from the practice of learning that has been in progress by the teacher (Yeni, 2011).

Idris (2011) suggested that learning of geometry is not easy and some students fail to develop an understanding of the concept of geometry, geometric reasoning and skill to solve the problems of geometry. Furthermore, Idris stated that a number of factors that lead learning of geometry is difficult which they are language of geometry, visualization and learning abilities are less effective for the low mastery of facts, concepts and principles of geometry.

According to Soerjono, one of among the causative factor is the intellectual ability of students (Suyanto, 2005). The results of Burger and Shaughnessy's study (1986) demonstrated that the intellectual ability of students plays an important role in the mastery of facts and concepts of geometry.

Intellectual abilities are spatial ability and auditory ability which are very close relationship with the cognitive aspects of students in general. The studies showed that the understanding of spatial knowledge can affect the performance related to academic tasks especially math, reading and science (Tambunan, 2006). Furthermore, according to Piaget and Inhelder (Tambunan, 2006) stated that spatial ability which was an aspect of cognition that develops in line with the cognitive development.

Piaget described the sequence into four stages of cognitive development which is qualitatively different, namely: (1) the stage of sensory motor (2) pre-operational stage, (3) the concrete operational stage, and (4) the formal operational stage (Hergenhahn, 2009). Piaget claimed that all children pass through four stages with different speeds, but none of the children who passed through one of the four stages (Monks, 2006; Yazdani, 2008). Piaget suggested that students should use logical operations to get structure of knowledge and their changes (Leongson & Limjap, 2003). The more often children move and find new things, the children will increasingly have new schemes that are used to develop their logical operation.

One of stages of cognitive development is formal operational stage. Sawyer et al, Dickinson and Lee classified into two of the formal operation. They are early formal operations and late formal operations (Adey, 1995). Students at the early formal have a good effort to solve the problems by using all its logical operations. The resulting solution is correct but there are little mistakes in using a type of logical operation. They can predict the final answers so that any data and information geared towards achieving that goal.

Students at the late formal is able to answer correctly. They show the use of logical operations well. They relate the data and information to resolve. If there is an impasse in resolving a problem they can find another alternative. Students demonstrated a broad understanding of the problem, using a cognitive schema to build understanding of the structure of the problem, as well as choosing the right strategy.

By using the scores as a basis and paying attention to the level of understanding of the students showed in solving problems, level of cognitive development of students based on tests of Piaget's logical operations classified as shown in **Table 1**.

Table 1. Level of Students' Cognitive Development Based on The Test of Piaget's Logical Operation

Interval Score	Level of Students's Cognitive Development
0-35	Early Concret
36-70	Late Concret
71-105	Early Formal
106-140	Late Formal

(Adopt from Loengson & Limjap, 2003)

METHOD

This research paper is exploratory descriptive study with data analysis by qualitative approach which main data in the form of words that are linked into sentences. Qualitative methods is chosen to investigate and to profile students' understanding in constructing the relationship of quadrilateral based on their geometric thinking. Moreover, profile of student's understanding is natural background and the main instrument is the researcher's own research. It means that the data which is analyzed in form descriptive and not in the form of figures as well as in quantitative research.

This research was conducted in 2 secondary schools in the area of Kabupaten Bone, Makassar, Indonesia. The participants in this research paper are two students eighth-grade secondary shool. The reason for choosing

the participants is they are at the stage of formal operations, so that the participants are able to think more abstractly, capable of inductive and deductive thinking and be able to think logically.

To determine the stage of cognitive development of participants who are at the early and late formal operation stage, then it is used the test instrument of Piaget Logical Operations (Piaget TOL) developed by Leongson and Limjap (2003). This instrument is a description form which consists of 35 questions. By referring to the opinions of Schoenfeld, Loengson and Limjap, researchers determined the level of cognitive development of students based on Piaget's logical operation test. If the scores obtained by participant 71-105, then the level of cognitive development of students at the level of early formal operation. In addition, if the particpants' scores 106-140, then the level of cognitive development of students at the level of late formal operation.

To obtain valid data in this study, then it was done the validation of data. One of the qualitative research validation procedures that can be performed is by means of triangulation. Validation of the data in this way is done by repeatedly checking with different time. Sugiyono (2005) called the data validation process by triangulation of time.

In this study the data analyzed consisted of the results of task-based interviews related to participant's geometric thinking in understanding the relationship between the quadrilateral that was done at home and school. The form of task-based interviews provided consists of several activities. These activities are as follows.

1. **Drawing quadrilateral:** In this activity, the participants are asked to draw as many quadrilaterals as they know.
2. **Identifying quadrilateral:** In this activity, the participants are asked to identify and recognize characteristics and definition of quadrilateral. Furthermore, the participants are asked to arrange the relationship of the characteristics of two quadrilaterlas based on the quadrilateral provided and ask the participants to state the reason.
3. **Making a diagram related to the relationship of quadrilateral:** In this activity, the participants are asked to make a diagram of the relationship between the quadrilaterals based on the definition of quadrilateral made.

RESULTS

The results of task-based interview from the participants on data collection that illustrate the partcipants' understanding toward the concept of quadrilateral by drawing, identifying and making relationships between quadrilateral using a diagram will be described as follows.

The Result of Investigating of the Actvities of the Participant at the Stage of Early Formal Operation (PEFO)

Drawing quadrilateral

In drawing quadrilateral, the participant at the stage of early formal operations (PEFO) can draw variety of different quadrilateral shapes are infinite by paying attention to the attributes of the shape, the size and position of quadrilateral.

More clearly on task-based interviews, the activities of the participant (PEFO) when drawing a quadrilateral, the participant (PEFO) drew quadrilateral by paying attention to characteristics or attributes of quadrilateral drawn as in **Figure 1**.

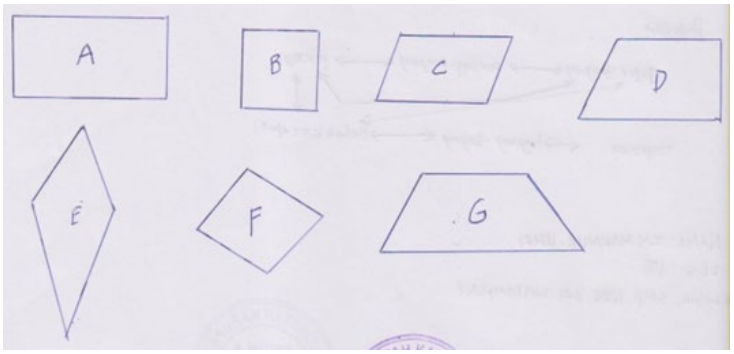

Figure 1. Quadrilaterals by PEFO

Drawing quadrilateral

In the activity of identifying different quadrilateral, the participant (PEFO) paid attention two attributes of quadrilateral based on size of the side and position While, for identifying the same quadrilateral, the participant attentioned attribute number of pairs of opposite sides and parallel, the angle of sight, the existence of a right angle which was owned by the rectangle and the size of both the adjacent sides or opposite sides and parallel. Definition of quadrilateral which was made by the participant (PEFO) accurate, excessive and inaccurate depending on the definition of quadrilateral used as a reference in defining a quadrilateral.

Clearly explained that the particpant (PEFO) made inferences when:

a. Identifying Parallelogram

1. PEFO identified several different models by taking into the attributes of size of parallelogram and the position of parallelogram.
2. PEFO identified some of the same characteristics of parallelogram by regarding the attribute size of side and the size of the angle. They were parallel and opposite sides had the same length and do not form a right angle.
3. Referring to the definition of parallelogram was a quadrilateral of which two pairs of parallel opposite sides, or two pairs of opposite sides of equal length, or a pair of parallel opposite sides of the same length, then the attributes given by subject to construct a definition parallelogram was excessive.

b. Identifying Rectangle

1. PEFO identified several different rectangular models by taking the atribute of size and the position of rectangle.
2. PEFO identified a common characteristic of some rectangles that four corners was right angle, opposite and parallel sides equal in length.
3. Referring to the definition of a rectangle was a parallelogram whose one of the corners was right-angled, the attributes given by PEFO to construct a definition of the rectangle is inaccurate.

c. Identifying rhombus

1. PEFO identified several rhombus in different models with regard to the attributes of size and position of the shape.
2. PEFO identified a common characteristic of some rhombus that all sides of rhombus had the same length, opposite parallel sides and equal in length, did not have the right angle.
3. If the definition refers to a rhombus was a quadrilateral whose four sides the same length as the attributes of a given by PEFO to build rhombus definition was accurate.

d. Identifying square

1. PEFO identified some square in different models with regarded the zise and position of the shape.
2. PEFO identified the characteristics of some models of the same square i.e., all sides is equal in length, parallel opposite sides and equal in length and all angels were right angel.

3. If the definition refered to the square was rhombus which one of its angles was right angel or quadrilateral whose four sides the same length and the angle is right angel, then the definition given the participant was accurate.

e. Identifying Trapezoid

1. PEFO identified several trapezoids in different models with regard to side length and size attributes, position and the kinds of trapezoid.
2. PEFO identified the characteristics of some models of trapezoids which had one pair of parallel opposite sides and the parallel sides of unequal length.
3. If the definition refered to the trapezoid was a quadrilateral having parallel opposite sides or rhombus which had only a pair of parallel opposite side, then the definition given the participant was accurate.

f. Identifying Kites

1. PEFO identified several kites in different models with regard to size of the kites and position of kites
2. PEFO identified the characteristics of some models of kite was adjacent sides of the same length and the angle confronted was same large.
3. If the definition refered to a kite was quadrilateral that had two pairs of adjacent sides of the same length and the sides did not overlap, then the definition given by the participant was accurate.

Making a diagram related to the relationship of quadrilateral

In the activities of making diagram of relationship between quadrilateral, the participant (PEFO) made diagram relationships between quadrilateral as in **Figure 2**.

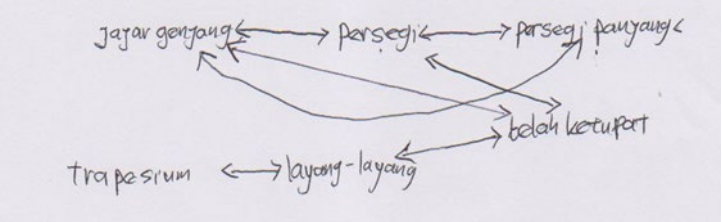

Figure 2. Diagram of the Relationship Between Quadrilateral Made by PEFO

Based on **Figure 2**, the participant (PEFO) explained that there were 21 possible relationships between quadrilateral parallelogram, rectangle, rhombus, square, kite, and trapezoid. Of these 21 relationship, there were only 17 possible relationships, it was caused by definition trapezoid was a quadrilateral which had a side that only a pair of parallel sides. PEFO recognized 4 accurate definitions from 6 accurate possible definitions. PEFO made 10 analytical definitions of 10 analytical definitions probably and 8 of them were accurate.

In simple terms, the explanation made by the subject related to the relationship between the quadrilaterals can be described as in **Figure 3**.

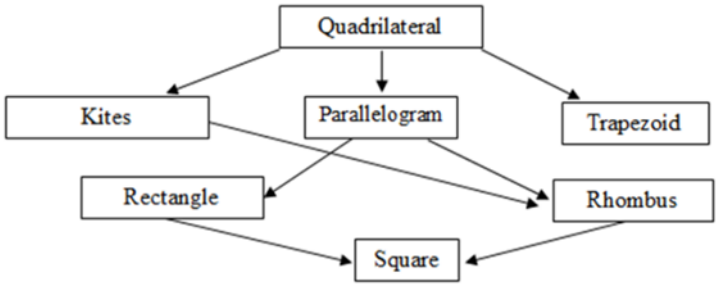

Figure 3. Relationship of Various Types of Quadrilateral by PEFO

The Result of Investigating of the Actvities of the Participant at the Stage of Late Formal Operation (PLFO)

Drawing quadrilateral

In drawing quadrilateral, the participant at the stage of late formal operations (PLFO) can draw variety of different quadrilateral shapes were infinite by paying attention to the attributes of the shape, the size and position of quadrilateral.

More clearly on task-based interviews, **the activities of the participant (PLFO) when drawing a quadrilateral**, the participant (PLFO) drew quadrilateral by paying attention to characteristics or attributes of quadrilateral drawn as in **Figure 4**.

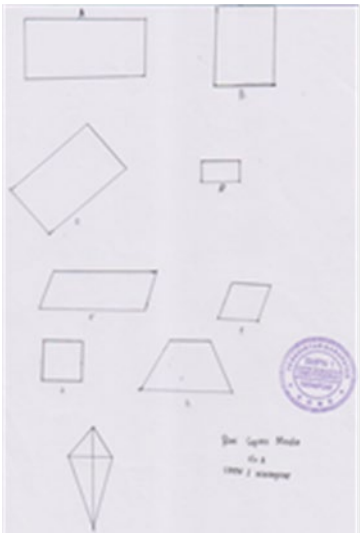

Figure 4. Quadrilaterals by PLFO

Identifying quadrilateral

In the activity of identifying different quadrilateral, the participant (PLFO) paid attention three attributes of quadrilateral based on size of the side, kind of quadrilateral (trapezoid) and position While, for identifying the same quadrilateral, the participant attentioned attribute number of pairs of opposite sides and parallel, the angle of sight, the existence of a right angle which was owned by the rectangle and the size of both the adjacent sides or opposite sides and parallel. Definition of quadrilateral which was made by the participant accurate and inaccurate depending on the definition of quadrilateral used as a reference in defining a quadrilateral.

Clearly explained that the participant made inferences when:

a. Identifying Parallelogram

1. PLFO identified several different models by taking into the attributes of size of parallelogram and the position of parallelogram.
2. PLFO identified some of the same characteristics of parallelogram by regarding the attribute size of side and having two pairs of opposite sides parallel and opposite sides of the same length.
3. Referring to the definition of parallelogram was a quadrilateral of which two pairs of parallel opposite sides, or two pairs of opposite sides of equal length, or a pair of parallel opposite sides of the same length, then the attributes given by PLFO to construct a definition parallelogram is accurate.

b. Identifying Rectangle

1. PLFO identified several different quadrilateral models by taking the atribute of size and the position of rectangle.
2. PLFO identified a common characteristic of some quadrilaterals that four corners was right angle, opposite and parallel sides equal in length.

3. Referring to the definition of a rectangle was a parallelogram whose one of the corners was right-angled, the attributes given by PLFO to construct a definition of the rectangle was inaccurate.

c. Identifying rhombus

1. PLFO identified several rhombus in different models with regard to the attributes of size and position of the shape.
2. PLFO identified a common characteristic of some rhombus that all sides of rhombus had the same length, opposite angle was equal.
3. If the definition refered to a rhombus was a quadrilateral whose four sides the same length as the attributes of a given by the participant to build rhombus definition was accurate.

d. Identifying square

1. PLFO identified some square in different models with regarded the zise and position of the shape.
2. PLFO identified the characteristics of some models of the same square i.e., all sides was equal in length and becomes right angel, opposite sides were parallel and equal in length.
3. If the definition refered to the square was rhombus which one of its angles was right angel or quadrilateral whose four sides the same length and the angle was right angel, then the definition given the participant was inaccurate.

e. Identifying Trapezoid

1. PLFO identified several trapezoids in different models with regarded to side length and size attributes, position and the kinds of trapezoid.
2. PLFO identified the characteristics of some models of trapezoids which had one pair of parallel opposite sides and the parallel sides of unequal length.
3. If the definition refered to the trapezoid was a quadrilateral having parallel opposite sides or rhombus which had only a pair of parallel opposite side, then the definition given the participant was accurate.

f. Identifying Kites

1. PLFO identified several kites in different models with regard to size of the kites and position of kites
2. PLFO identified the characteristics of some models of kite was adjacent sides of the same length and opposite sides of unequal length, opposite angle was equal.
3. If the definition refered to a kite was quadrilateral that had two pairs of adjacent sides of the same length and the sides did not overlap, then the definition given by participant was accurate.

Making a diagram related to the relationship of quadrilateral

In the activities of making diagram of relationship between quadrilaterals, the participant (PLFO) made diagram relationships between quadrilaterals as in the following **Figure 5**.

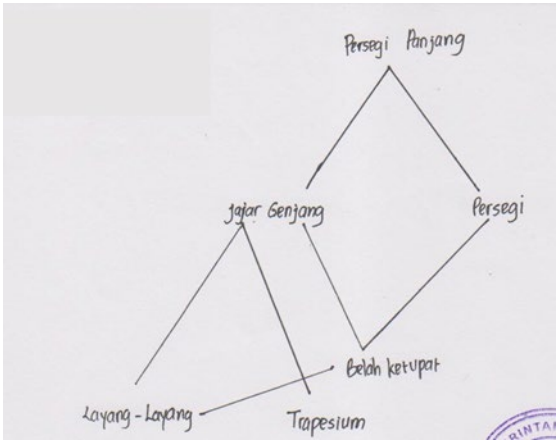

Figure 5. Diagram of the Relationship Between Quadrilateral Made by PLFO

Based on **Figure 5**, the participant (PLFO) explained that there were 21 possible relationships between parallelogram, rectangle, rhombus, square, kite and trapezoid. Of these 21 relationship, there are only 12 possible relationships, this is because the definition of trapezoid is a quadrilateral that has only a pair of parallel sides. The participant recognized the 2 accurate definitions of 6 possible accurate definitions. The participant made 8 analytical definitions of the 8 possible ones and 6 of them were accurate.

In simple terms, the explanation made by the subject related to the relationship between the quadrilaterals can be described as in **Figure 6**.

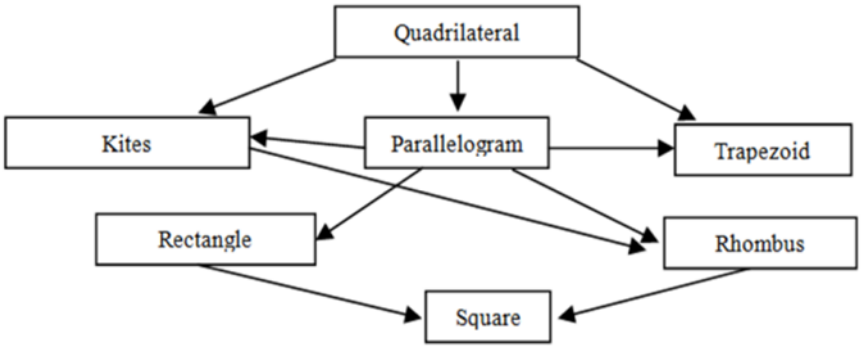

Figure 6. Relationship of various types of quadrilateral by PLFO

DISCUSSION

This research paper was to investigate the students' ability in understanding the relationship of quadrilateral based on their geometric thinking skill reviewed from their cognitive development. From the results of this research paper, it can be explained that the participant who is at the early formal operation (PEFO) made a definition based on the relationship between quadrilaterals. If it is viewed from the standpoint of analytical, the definition made by participant (PEFO) presents relationship 14 relationships between quadrilateral possible. Shrinkage occurs relationship of 15 who may be 14 possible relationship. This shrinkage occurs because there is a genus that are used but not genus proksimum.

Based on the understanding which is recognized by the participant (PEFO), trapezoid is a quadrilateral which has a pair of parallel sides and a kite is a quadrilateral which is two pairs adjacent sides has the same length, then these results can be interpreted that the subject makes sense analytically.

Furthermore, the participant (PEFO) concludes that there are 21 possible relationships between quadrilateral parallelogram, rectangle, rhombus, square, kite, and trapezoid. From 21 of this relationship, there are only 17 probably connections which is made by the participant (PEFO), it is caused by definition trapezoid is a quadrilateral which has a side that only a pair of parallel sides. The participant made 10 analytical definitions of 10 analytical definitions probably and 8 of them are accurate.

While the participant who is at the late formal operation (PLFO) makes a definition based on the relationship between quadrilaterals. If it is viewed from the standpoint of analytical, the definition made by the participant (PLFO) presents relationship 7 relationships between quadrilateral possible. Shrinkage occurs relationship of 7 who may be 15 possible relationship. This shrinkage occurs because there is a genus that are used but not genus proksimum.

Based on the understanding which is recognized by the participant (PLFO), trapezoid is a quadrilateral which has a pair of parallel sides and a kite is a quadrilateral which is two pairs adjacent sides has the same length, then these results can be interpreted that the participant (PLFO) makes sense analytically.

Furthermore, the participant (PLFO) makes a conclusion that there are 21 possible relationships between quadrilateral parallelogram, rectangle, rhombus, square, kite, and trapezoid. From 21 of this relationship, there are only 12 probably connections which is made by the subject, it is caused by definition trapezoid is a quadrilateral which has opposite sides parallel and equal in length. The participant (PLFO) recognized 2 accurate definition of 6 accurate definition which might and subject made 8 analytical definitions of 8 analytical definitions probably and 6 of them are accurate.

Based on the explanation before, the findings of this research paper confirmed the result of research conducted by Van de Walle (2001) that, in the level informal deduction, students can already see the relationship between the properties of a geometry and the properties between several geometric shapes.

Students can make abstract definitions, find the properties of various constructs using informal deduction, and can classify structures in a hierarchical manner.

The two participants has the abilities to carry out operations that state the relationship between relationships, understand the concept of promotion. The characteristic of the participants in this research have a ability to do mortgage-deductive reasoning, namely the ability to compile a series of defintion based on their opinion and their experience. It is in line with the results of Solso's research (2008) that on this level children's cognitive structure has been able to use symbols, ideas, abtraction and make generalization.

Both participants solve the problem given using classification skills, proportions or ratios, probabilities and correlations. This is in accordance with the results of Loengson & Limjap's research (2003) stating that "*the logical namely: classification, seriation, logical multiplication, compensation, proportional thingking or ratio, probability and correlation thingking can be used as cognitive tools in mathematicial problem solving"*

It is indicated that it is required logical operation to solve the problem. Furthermore, logical operation is needeed to describe the level and progress of reasoning of learners, the concept of reasoning pattern can be used. Instead of dealing with the cognitive levels or logical operation of intellectual development, identifiable and reproducible thought progress directed at kind of task can be observed (Loengson & Limjap, 2003).

In this research, the researcher explore also student's thinking processes and student's understanding in making the relationship between the quadrilateral. Some of these students' activities can be easily identified by students' words or actions and can involve a broader analysis of tasks.

The participant at the early formal solving given problem is able to show the relationship between the concept or idea that shows how the problem is solved despite minor errors in representation, strategies and connectedness done while the participant at the late formal operations solving problems is able to choose and use the right strategic solution and achieve the right answer well.

CONCLUSIONS

From the results of this research paper, in general student's understanding who is at the stage of early formal operations and late formal operation can understand the relationship between quadrilateral well but is less able to pay attention or identify the relationship both similarities and differences of quadrilateral. Therefore, the researchers suggested that the educators must pay attention the stage of cognitive development of students in the learning process, particularly in understanding the relationship between quadrilaterals.

On the other hand, in the activities of drawing quadrilateral. There are students's tendency, in this study, to draw a quadrilateral by starting from images that is very familiar for the participants or often encountered and recognized by the participants as a rectangle and a square. Likewise, when the participants made diagram relationship between quadrilaterals, the tendency was happening again. The participants tended to start from rectangle and connect it to the square. This indicates that the learning process in schools especially for quadrilateral, teachers often taught students ranging from rectangle or square so that students are only very familiar with both of them. Therefore, researchers also suggest to educators to teach not only the quadrilateral from a square or rectangle but start another quadrilateral such as parallelogram, rhombus, kite and trapezoid.

ACKNOWLEDGEMENTS

I would like to express my deepest appreciation to all those who provided me the possibility to complete this report. A special gratitude I give to chairman of LLDIKTI Wilayah IX, Kemeristekdikti Indonesia, Prof. Dr. Jasruddin, M.Si., whose contribute in providing research assistance so that this research can work well and the researcher can write this report.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Agustan Syamsuddin – Mathematics Education Department and Magister of Elementary of Education Postgraduate Program,Universitas Muhammadiyah Makassar, Indonesia.

REFERENCES

Abdullah, A. H., & Zakaria, E. (2011). Students' Perceptions Towards the van Hiele's Phases of Learning Geometry Using Geometer's Sketchpad Software. *Australian Journal of Basic and Applied Sciences*, *5*(7), 787-792

Abdusakkir. (2002). Pembelajaran Geometri Berdasar Teori van Hiele Berbantuan Komputer. *Jurnal Matematika atau Pembelajarannya*, *8*(special edition), 344-348.

Adey, P. (1995) *Cognitive acceleration: Science and other entrances to formal operations*. London: King's College London.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the Van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, *17*(1), 31-48. https://doi.org/10.2307/749317

Hergenhahn, B. R., & Olson, M. H. (2008). *Theories of Learning*. Jakarta: Kencana.

Husnaeni. (2006). Penerapan Model Pembelajaran van Hiele dalam Membantu Siswa Kelas IV SD Membangun Konsep Segitiga. *Jurnal Pendidikan, 7*(2), 67-68.

Idris, N. (2011). The Impact of Using Geometers' Sketchpad on Malaysia Students' Achievement and van Hiele Thinking. *Journal for Mathematics Education*, *2*(2), 94-107.

Leongson, J. A., & Limjap, A. A. (2003). *Assessing the Mathematics Achievement of College Freshmen using Piaget's Logical Operations*. Waikiki: Hawaii International Conference on Education.

Monks, F. J. A. K. (2006). *Psikologi Perkembangan: Pengantar dalam Berbagai Bagiannya*. Yogyakarta: Gadjah Mada University Press.

Solso, O. H. (2008). *Psikologi Kognitif*. Jakarta: Erlangga.

Sugiyono. (2005). *Memahami Penelitian Kualitatif*. Bandung: Alfabeta.

Sunardi. (2005). *Pengembangan model Pembelajaran Geometri Berbasis Teori van Hiele* (Dissertation of Mathematics Education Study Program), Universitas Negeri Surabaya.

Suyanto. (2005). *Penelusuran Tahap Berpikir Geometris van Hiele Siswa Kelas III SMP Negeri 21 Surabaya yang Mengikuti Bimbingan Belajar Primagama pada Pokok Bahasan Segiempat* (Thesis Mathematics Education Study Program), Universitas Negeri Surabaya.

Tambunan, S. M. (2006). Hubungan Antara Kemampuan Spasial dengan Prestasi Belajar Matematika. *Makara, Sosial Humaniora, 10*(1), 27-32. https://doi.org/10.7454/mssh.v10i1.13

Van de Walle, J. A. (2001). *Geometric Thinking and Geometric Concept. In Elementary and Middle School Mathematics. Teaching developmentally 4th ed.* Boston: Pearson Education.

Yazdani, M. A. (2008). The Gagne-van Hiele Connection: A Comparative Analysis of Two Theoretical Learning Frameworks. *Journal of Mathematics Science & Mathematics Education, 3*(1), 58-63.

Yeni, E. M. (2011). Pemanfaatan Benda-Benda Manipulative Untuk Meningkatkan Pemahaman Konsep Geometri Dan Kemampuan Titik Ruang Siswa Kelas V Sekolah Dasar. *Jurnal Pendidikan Matematika, 1*(special edition), 63-75.