LOOK ACADEMIC PUBLISHERS
OPEN ACCESS

INTERNATIONAL JOURNAL OF ENVIRONMENTAL & SCIENCE EDUCATION
2016, VOL. 11, NO.14, 7278-7285

Exploration of Mathematics Problem Solving Process Based on The Thinking Level of Students in Junior High School

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ABSTRACT

Several studies suggest that most students are not in the same level of development (Slavin, 2008). From concrete operation level to formal operation level, students experience lateness in the transition phase. Consequently, students feel difficulty in solving mathematics problems. Method research is a qualitatively descriptive-explorative research aimed at comprehending the process of mathematics problem solving based on students' thinking level. Formal subject described in a structured manner so that there is no information that eliminated in the calculation process. While in transition subject, information which is constructed is only based on empirical knowledge. And on a concrete subject, thinking process can directly determine the solution of a problem. Students in formal thinking level are able to plan a problem solving by relating an information that is obtained to an information which is logically asked. Transitional thinking level are able to visualise the problems logically when the context of the problems are closely related to the experience they have. And concrete thinking level is only able t plan a problem solving when the problem can be immediately and easily analysed.

KEYWORDS
problem solving, logical level, formal subject, transition subject, concrete subject

ARTICLE HISTORY
Received 01 September 2016
Revised 06 September 2016
Accepted 09 September 2016

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Introduction

Mathematics as a compulsory subject both in elementary school and junior high school has strategic role in establishing formal knowledge characters for students. Having good ability in doing mathematics operation, being skillful in solving problems, and being critical in interpreting non-routine problems are general prerequisite to have a good formal reasoning. Those things are closely related to mathematics taught in school both in elementary level and secondary level, which is periodically appropriates to cognitive development level Piaget (Ayriza, 1995). The mathematics materials are arranged hierarchically by considering the aspect of students' cognitive development to make an optimum learning process (Suherman, 2001). However, practically, most students feel difficulty in understanding mathematics concepts. Whereas, by looking the fact that the mathematics materials have been well arranged based on Piaget's cognitive development theory, the students should have understood the lessons.

Several studies suggest that most students are not in the same level of development (Slavin, 2008). From concrete operation level to formal operation level, students experience lateness in the transition phase. Consequently students feel difficulty in solving mathematics problems.The development lateness are influenced by several factors. One of them is the model of teaching in school. Most schools both elementary school and junior school put less attention to students' level of thinking. Much worse, there is a compulsion of conceptual understanding to students.

The weakness of students in developing reasoning ability has effect on their abilities in solving problems. Piaget suggested that students are ready to develop concept or material when they have necessary scheme meaning that learning process of students is impended when the formal reasoning of students is not appropriate to material which is taught (Nuroso & Siswanto, 2009). A Meaningful learning process is not only how to make students come up with concept in their minds but also how to make them skillful in analyzing and solving problems.The learning process, which is implemented by most schools, refers to the assumption of direct information processing. Whereas, in mathematics learning, students need many adaptations before mastering an advance cognitive skill.

The rapid development of knowledge stimulates teachers to prepare their students to have high level of competitiveness in global life.The skill of problem solving is a primary point needed by students to realize the importance of mathematics in daily life. In addition, in problem solving, students are encouraged to develop his formal reasoning independently, free from several conservative paradigms, to manage their thinking. several routine procedures and problems involving static learning process will be automatically left by students since it is not interesting. Meanwhile, in problem solving, activity of managing thinking effectively, efficiently, and flexibelly is strongly emphasized. Therefore, the author aimed at explorating the process of mathematics problem solvingbased on students' thinking level in SMP Negeri 2 Galesong Selatan.

Literature Review

According to Michalewicz and Fogel (2004), a problem refers to a situation in which there is a difference between fact and will. Consequently, it forces a person to utilize his potential in order to reduce that gap. Meanwhile, Hoosain (2003) defined a problem, according to Kantowski (1977), Mervis (1978), and Buchanan (1987), as a non-routine problem, which has not common procedure and algorithm to solve, requiring thought to find a useful information to get the solution.

Process of Mathematics Problem Solving

Problem solving is a complex mental process, involving visualization, imagination, abstraction, and assosiation of information. Therefore, problem solving through mathematics learning process can help students increase and develop their abilities in the aspect of application, analysis, synthesis, and evaluation (Anderson & Krathwohl, 2001). The process of problem solving is a complex cognitive process. Further, Winkel (2007) said that in the term of information processing, one is said to have problem, when he has goal, but there has not yet a “tool” to achieve the goal.

In the recent decades, many experts have been developing model of problem solving process, especially in mathematics education.The development is based on an assumption that problem solving skill is abstract and can be transfered in problem solving with different context.One of the examples of general problem solving model was developed by Bransford and Stein (Suharnan, 2005) consisting of (1) identifyingproblem identification, (2) defining problem through thinking process about the problem and selecting relevant information, (3) explorating possible solution and doing verification from several perspectives, (4) implementing the selected strategies, and (5) reviewing and evaluating the result obtained from the strategy implementation.

Polya (1973) defined problem solving as an effort in finding solution of a problem to achieve a goal that seems difficult to gain. According to Polya, problem solving in mathematics encompasses 4 steps, namely (1) understanding problems, (2) planning the steps in solving the problems, (3) implementing the strategies to solve the problems, and (4) doing verification.

The level of the difficulty and the ability in a process of problem solving is determined by several factors. According to Suharnan (2005), there are several factors that can influence the level of difficulty of a problem, namely (1) problem understanding, (2) mental representation, (3) the coverage of problem, and (4) problem imbalance.

Thinking Level. In general, thinking is assumed as one of cognitive processes that can’t be physically seen, which in th form of mental activity to obtain knowledge. Piaget (Suharnan, 2005) divides cognitive development into four levelsnamely, motoric sensory level (year 0 – 2), pre-operational level (year of 2 – 7), concrete operation level (year of 7-11), and formal operation level (more than 11 years). Each level of development has its own characteristics and, consequently, the thinking framework of each level is also different.

According to Piaget (Hergenhahn & Olson, 2008), every child has an organized response system, which is called scheme (plural form: schemes). The development of children causing the increase of the number of schemes in certain period of time is called cognitive structure.To create an interaction between a child and his circumstance, there should be a cognitive structure or information absorption from the circumstance to cognitive structure.

Cognitive structure, developing from infant to child, was defined by Piaget as stage of motoric sensory. Further, gradually, pre-operational stage will be replaced by more logic thinking structure, called concrete operation level and formal operation level. According to Piaget as cited in Gredler (1992) operation refers to cognitive structre organizing logical reasoning in wide perspective.In addition, operation is defined as a thinking activity that can execute complex and dynamic tasks.Each individual who is able to apply an operation in a thinking process can think not only linearly but also regressively. Moreover, his ability in considering the change of a form is not restricted by sense.

In teenage, a logic thinking process has important role in solving problem. According to Piaget (1965), there are three stages of thinking level in teenage, namely:

Concrete operation thinking level. One of important tasks learned by students in this level is arrangement, in other words, arranging objects in good sequence. To be able to do that thing, students should be able to classify objects based on appropriate criteria.Besides that, students, in concrete operation level, move from egocentric thinking to not centrist or objective thinking.An objective thinking is likely to make a student consider other students having different perspective with him (Slavin, 2008). According to Anderson as cited in Suherman (2001), identified that there are six kinds of eternity in the form of conservation reasoning developing in the stage of concrete operation, namely; (1) eternity of numbers (year of 6-7), (2) eternity ofmaterials (year of 7-8), (3) eternity of length (year of 7-8), (4) eternity of width (year of 8-9), (5) eternity of mass (year of 9-10), and (6) eternity of volume (year of 11-12).

Transitional thinking level. Students in this level can think abstractly from their empirical experience. The dependence of concrete objects of students restricts them in understanding and manipulating the relationship among abstractions because their understanding can't reach a representation which can't be directly recognized (Ausubel & Ausubel, 1966).
According to Tall (2008), based on thinking of a person,mathematics domain is divided into three parts; (1) conceptual world, (2) symbolic perceptual world, and (3) formal axiomatic world. In the scope of mathematics learning in school, mathematics is viewed as conceptual world.In other words, most students view mathematics world as perception and reflection toward observable and imaginable objects. Thereofore, students in this level view mathematics as their reflective perception.

Formal operation thinking level. Students who are able to think formally are not dependent to concrete objects to solve a problem.They are also able to develop their abilities of reasoning and thinking so they are skillful in utilizing symbols, ideas, abstractions, and generalization forms.In addition, they are able to associate informations, create ideas, and solve problems.In addition, they can think like scientist and systematically do verification.

The main characteristic of thinking in formal operation stage is an ability to think abstractly. Meanwhile, according to Flavell as cited in Ayriza (1995), the characteristics of thinking formally are (1) being able to think in many possibilities, (2) hypothetical deductive thinking, (3) scientifically inductive thinking, (4) reflectively abstract thinking, (5) inter-proportional thinking process, (6) being able to understand the concept of permutation and combination, (7) being able to do an inverse and compensation, and (8) consolidation and solification of cognitive structure.

Logical Reasoning. In the level of concrete operation thinking and formal operation thinking, students are capable of developing logical reasoning in solving problems. According to Piaget as cited in (Fah (2009) there are five kinds of reasoning in the level of formal operation thinking including conservation reasoning in which there is a consolidation and solidification process. Consequently, it can be considered that, there are six logical reasonings in students' thinking in both concrete operation level and formal operation level, namely; (1) conservation reasoning, (2) proportionalreasoning, (3) probability reasoning, (4) variable controlling, (5) correlational reasoning, dan (6) combinatorial reasoning.

Thinking is not an ampirically measurable activity. Instead, it just can be abstracted through the activities resulted from a thinking.There are several instruments developed by experts to identify the levels of thinking based on the logical reasoning of students (Kamaruddin, Abu Bakar, Surif, & Li, 2004) namely:

a. *Group Assesment of Logical Thinking* (GALT) test developed by Roadrangka (1983).
b. *Test of Logical Thinking* (TOLT) developed by Tobie and Capie (1981).
c. *Classroom Test of Scientific Reasoning* (CTSR) developed by Lawson (2000).

Method

The present research is a qualitatively descriptive-explorative research aimed at comprehending the process of mathematics problem solving based on students' thinking level. The subject of the research was a group of students grade IX in SMP Negeri 2 Galesong Selatan. The subjects consist of one formal subject, one transitional subject, and concrete subjects. The choice of the class was done using purposive technique. Meanwhile, the selection of the subjects was based on snowball technique. The process of the selection was done using three standardized diagnostic tools.The subjects which have been consistently determined through the tools were verified based on the category of thinking level. The technique of the data collection used in the research comprehends oftest andinterview. The data analysis used the categories of thinking level developed by some experts (Rodrangka 1983; Tobie & Capie, 1981; Lawson, 2000). Meanwhile, the data obtained from the test of problem solving and the interview were analyzed using three steps of qualitative data analysisnamely; (1) data reduction stage, (2) data presentation, and (3) data verification.

Result and Discussion

According to Polya (1973), the process of mathematics problem solving aregenerally divided into four stages, (1) understanding the problem, (2) devising problem solving, (3) carrying out problem solving according to a set plan, and (4) re-examining the solutions that have been obtained. Each stage has an indicator and hierarchy implementation. However, the process of mathematics problem solving can't be separated from two important factors, namely (1) problem characteristics, and (2) cognitive maturity. Therefore, the process of mathematics problem solving posed by Polya haven't to be absolutely followed by every student in sequence.

In this regard, the problem solving process adopted by each subject is associated with a given problem at oppurtuinity subject,there are several steps of different with Polya problem solving sequence. Problem solving steps taken by each subject in general different from each other. It's caused of cognitive maturity from each subject and mindset in managing the problem causing the steps have been take also different. In addition, the context of given problem also affects the mental representations that can be managed by each subject, so that different from the sequence of mathematics problem solving process that proposed by Polya.

In the process of understanding the problem, first construct a formal subject matter and determine what information is needed to solve the problem. The information described in a structured manner so that there is no information that eliminated in calculation process. I carrying out computing process, a formal subject involves analysis process and the ability of analyze a problem. While in transition subject, informationwhich is constructed is only based on empirical knowledge. Important information that can only be interpreted analytically is not included in the calculation process. Furthermore, at some more complex problems, the ways to analyze a problem is still influenced by intuitive perception. While on a concrete subject, thinking process can directly determine the solution of a problem. The information contained on the matter constructed simultaneously during problem-solving thinking of a plan . Concrete subject give the impression of haste in understanding a problem so that there are some important information that was not mentioned at all .

From the results shows that a formal subject plan for solving the problem by linking between information that has been obtained with information asked logically.Ability to think and analyze the situation of the various possibilities that are not only cause representation problem can be used consistent with the context problem. In addition , formal subjects has cognitive maturity to handle a variety of problem related to conservation reasoning , combinatorics , proportional , and variables control.While on the transition subject, the ability to visualize the problem can only be performed logically if the problem context closely related to the empirical experience. Although the transition subject is able to think proportion of the two objects at once, but the transition subject has not been able to think the possibilities in a space of case. That is the reason why thought process is intuitively more dominated when facing context problems that completely new experienced .While on the concrete subject, problem -solving plan to link any information logically if the problems encountered can be considered directly. New context problems encountered can not be described logically and intuitive thought process heavily influenced. Visualization problems can only be done through the preparation of object row.

Formal subject is able to apply the concept of opportunity and understand how to determine sample point and sample space of case . In addition , formal subjects are also able to think of the many other possible opportunities that may occur at case. Able to apply the concept of conditional probability, but not fully understand the reasons behind the application of these concepts in problem solving .While transition subject is able to apply the oppurtunity concepts and understand determination of sample points if only the problem can be perceived in concrete .On problems that require more reasoning , the transition subject is difficult to develop an understanding in determining the sample point and sample space logically. Meanwhile, concrete subject apply opportunities concepts in implementing problem solving based on factual knowledge.In general, concrete subject did not understand axiom opportunities. It is indicated in determine opportunities on a more complex problem, a relation between sample points and sample space can not be interpreted logically.Concrete subject's ability to implement a formula opportunities purely a result of the process of delivering the facts without adequate reasoning process followed.

Formal subjects verify solution that has been obtained through a sequence of more concrete thought processes.In order to verify the solutions that have been obtained, the sample point and sample space traced probabilities chronologically. Inductive thinking process provides an alternative way for formal subjects to affirm their faith in solution.While the transition subject re-examine of a solution that has been obtained by interpreting the solution through a sequence of thought processes previously. In this case, transition subject did not show any transition alternative way to test a solution that have been obtained.Meanwhile, concrete subject only able to rethink the processes that have been executed without being able to provide corrections or comments on the results of those thoughts. In this case, concrete subject also did not indicate alternative way to test the solutions that have been obtained.

Conclusions

Based on the results and the discussion, it can be concluded that students in formal thinking level are able to plan a problem solving by relating an information that is obtained to an information which is logically asked. In addition, they are able to think many possibilities and to analyze several situations which cause the problem representation can be associated with problem context. Besides that, they have a strong cognitive ability to solve many problems concerning about reasonings of conservation, combinatoric, proportional, and variable controlling. Meanwhile, students in transitional thinkin level are able to visualize or to describe the problems logically when the context of the problems are closely related to the experience they have.Although they are capable in thinking the proportion of two objects, however, they are not able to think many possibilities in an event.In other words, the thinking process which is intuitively done is dominated by an activity of solving problems which is strongly related to the experience of the students.

Whereas, students in concrete thinking level are only able t plan a problem solving when the problem can be immediately and easily analyzed. However, they have trouble in logically analyzing the problems in which they experience it for the first time. In further, they are just able to visualize the problems through objects arrangement.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

Alimuddin. (2012). *Proses Berpikir Kreatif Mahasiswa Calon Guru Kreatif Dalam Pemecahan Masalah Matematika Berdasarkan Gender [Creative Thinking Process of Students Candidate Creative Teacher in Mathematical Problem Solving by Gender] (Unpublished doctoral dissertation).* Surabaya: PPs Unesa Surabaya.

Anderson, L. W., & Krathwohl, D. R. (2001). *Kerangka Landasan Untuk Pembelajaran, Pengajaran, dan Assesmen [Framework for Learning, Teaching and Assessment].* Yogyakarta: Pustaka Pelajar.

Ausubel, D. P., & Ausubel, P. (1966). Cognitive Development in Adolescence. *Review of Educational Research, 36*(4), 403-413. doi:10.3102/00346543036004403

Ayriza, Y. (1995). Teori Perkembangan Kognitif Piaget Sebagai Alat Bantu Petunjuk Dalam Pelaksanaan Pendidikan Dasar 9 Tahun [Theory of Piaget's Cognitive Development As A Tool Indicator in Implementation of Basic Education 9 Years]. *Cakrawala Pendidikan*, 155-163.

Fah, L. Y. (2009). Logical thinking abilities among form 4 students in the interior division of Sabah, Malaysia. *Journal of Science and Mathematics Education in Southeast Asia, 32*(2), 161-187.

Gredler, M. E. (1992). *Learning and Instruction : Theory into Practice.* New York: Macmilan Publishing Company.

Hergenhahn, B. R., & Olson, M. H. (2008). *Theories of Learning.* (T. B. Wibowo, Trans.) Jakarta: Kencana Prenada Media Grup.

Hoosain, E. (2003). What are Mathematical Problems. *The Humanistic Mathematics Network Journal Online, 27*, 1-8. Retrieved from http://www2.hmc.edu/www_common/hmnj/hoosain.pdf

Kamaruddin, M. I., Abu Bakar, Z. B., Surif, J. B., & Li, W. S. (2004). Relationship Between Cognitive Styles, Levels of Cognitive Thinking And Chemistry Achievement Among Form Four Science Students. Retrieved from https://www.academia.edu/2649641/RELATIONSHIP_BETWEEN_COGNITIVE_STYLES_LEVELS_OF_COGNITIVE_THINKING_AND_CHEMISTRY_ACHIEVEMENT_AMONG_FORM_FOUR_SCIENCE_STUDENTS_PERHUBUNGAN_ANTARA_GAYA_PEMBELAJARAN_TAHAP_PEMIKIRAN_KOGNITIF_DAN_PENCAPAIAN_KIMIA_DI_KALANGAN_P

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. *Journal For research in mathematics Education, 8*, 163-180.
Karplus, R. (1977). Science Teaching And The Development Of Reasoning. *Journal of Research in Science Teaching, 14*(2), 169-175.
Lawson, A. E. (1978). Development and Validation of The Classroom Test of Formal Reasoning. *Journal of Research in Science Teaching, 15*(1), 11-24.
Lawson, A. E. (2000). *Classroom Test of Scientific Reasoning (Multiple Choice Version)* (Rev. ed.). Arizona: Arizona State University.
Michalewicz, Z., & Fogel, O. B. (2004). *How to Solve It: Modern Heuristics.* Berlin: Springer.
Nuroso, H., & Siswanto, J. (2009). Model Pengembangan Modul IPA Terpadu Berdasarkan Perkembangan Kognitif Siswa [Model Development of Integrated Science Books Based Cognitive Development Students]. *Jurnal Penelitian Pembelajaran Fisika, 1*(1), 35-46.
Ormrod, J. E. (2009). *Psikologi Pendidikan: Membantu Siswa Tumbuh dan Berkembang [Psychology of Education: Helping Students Grow and Improve].* (W. Indiarti, Trans.) Jakarta: Erlangga.
Piaget, J. (1965). *The Origins Of Intelligence In Children.* New York: International Universities Press, Inc.
Polya, G. (1973). *How to Solve It: A New Aspect Of Mathematical Method.* New Jersey: Princeton University Press.
Roadrangka, V., Yeany, R. H., & Padilla, M. J. (1983, April). The construction of a group assessment of logical thinking (GALT). *In 56th annual meeting of the National Association for Research in Science Teaching. Dallas, Texas, April*, pp. 5-8. Retrieved from http://kasetsartjournal.ku.ac.th/kuj_files/2010/A1009021418431093.pdf
Slavin, R. E. (2008). *Psikologi Pendidikan: Teori dan Praktik [Educational Psychology: Theory and Practice]* (Vol. I). Jakarta: Indeks.
Sugiono. (2011). *Metode Penelitian Pendidikan (Pendekatan Kuantitatif, Kualitatif, dan R&D) [Educational Research Methods (Quantitative, Qualitative and R & D Approach)].* Bandung: Alfabeta.
Suharnan. (2005). *Psikologi Kognitif [Cognitive Psychology].* Surabaya: Srikandi.
Suherman, E. (2001). *Strategi Pembelajaran Matematika Kontemporer [Contemporary Mathematics Learning Strategy].* Bandung: JICA UPI.
Tall, D. (2008). The Transition To Formal Thinking In Mathemathics. *Mathematics Education Research Journal, 20*(2), 5-24.
Tobin, K., & Capie, W. (1981). The Development and Validation of a Group Test of Logical Thinking. *Educational And Pshychological Measurement, 41*(2), 413-423. doi:10.1177/001316448104100220
Valanides, N. (1997, Desember). Formal reasoning and school achievement. *Studies In Educational Evaluation*, 169-185. doi:10.1016/S0191-491X(97)00011-4
Winkel, W. S. (2007). *Psikologi Pengajaran [Teaching Psychology].* Yogyakarta: Media Abadi.